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Λ-Hypernuclear States as Dihadronic Molecules

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ABSTRACT

The study of exotic hypernuclei attracts a great deal of interest in nuclear physics. The reality of heavy hyperon hypernuclei is the subject of intense concern among theoreticians and experimenters in recent years. The core-hyperon model uses to explain abnormal nuclei spectra, recent observations of new exotic heavy hyperon hypernuclei cannot be explained or predicted by ordinary heavy core nuclei. These exotic hypernuclei states are a two-cluster bound states. We calculate the mass spectrum and constituent mass of particles in hypernuclei using the relativistic Schrödinger equation with molecular pseudoharmonic-type potential between particles inside the core and hyperon. Such calculations represent the interaction between the hyperon and the nuclei core. I review recent theoretical studies on the ground states and the excited states of hypernuclei bound states. Finally, we present explicit predictions of the exotic bound states based on the interactions obtained from quantum field theory and the projective unitary representation model. Studies have shown that by increasing the mass number of hyperon-core states, the value of the constituent mass and energy eigenvalue of Λ-hypernucleus increases. Also, by growing and increasing the proton number in the (Λ-N) states the value of the constituent mass of Λ-hyperon increases.

INTRODUCTION

This thesis investigates the bound state of two hadrons, i.e., hadronic molecules. The current research is primarily concerned with the interplay between two neutral color states. The mass spectra and decay features of di-hadronic systems are discussed in this thesis using the prospective model framework, which has been shown to be a viable theoretical tool in the study of perturbative QCD (Quantum Chromo Dynamics). The study of hadronic compounds has been attempted for quite some time. The deuteron (the bonded state of a proton and a neutron) is the sole candidate who has a verified position in the hadronic molecular family, while additional members are still waiting to join. The hypothesized basic theory of the strong interaction, known as QCD, on the other hand, is a non-abelian gauge theory, and its immediate ramifications predicted intricate color-neutral states (exotic states) and vast numbers of hadronic molecules. The new experimental breakthroughs have produced a great amount of data and a big number of surprises during the previous two decades, as well as a number of novel states that cannot be described by the standard meson or baryon scheme. However, the discovered states are still awaiting confirmation of their unique sub-structure from both a theoretical and experimental standpoint, as well as assignment to a certain family such as glueball, hybrid, multiquark, or molecules. All of these states might have unconventional structures, the molecular interpretation of which is the focus of this thesis. The mass spectra of di-mesonic, meson-baryon, and di-baryon combinations have been determined using the hydrogen-like trial wave function. Di-hadronic compounds, like deuteron, are approximated as abounding systems. Various realistic or semi-realistic potentials have been devised for the study of the deuteron [1-5].

Identifying hadronic molecules and the relations among two color-neutral hadrons are two issues with the molecular sample of multi-quark systems. We investigated di-hadronic molecular structures using hypothesized interaction potentials such as s-wave one boson exchange potential and

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Screen Yukawa-like potential and came to the conclusion that the two color-neutral hadrons in a hadronic molecule had dipole-like relations. The current research is a continuation of our prior research [1]. The mass spectra of meson-meson and meson-baryon molecules are indicated using the hypothesized interaction potential.

A hypernucleus is a subatomic bound system containing at least one hyperon that can be used to study nuclear forces and baryonic interactions with up, down, and strange quarks. For over seven decades, hypernuclei have been researched extensively in reactions involving cosmic rays and accelerator beams. Experimental investigations of hypernuclei have progressed to a new level in recent years, thanks to explosive collisions of heavy-ion beams. However, this research has yielded two surprising results: the presence of a bound state of two neutrons with a Λ hyperon, and the lightest three-body hypernuclear system, the so-called hypertriton. Solving these difficulties will have ramifications not just for our knowledge of basic baryonic interactions with weird quarks but also for our understanding of the structure of neutron stars' deep interiors. In the past 20 years, there has been a surge in research into the characteristics of hypernuclei, both theoretically and empirically. One of the major goals of experiments is to obtain and study hypernuclei in order to better understand nuclear phenomenology.

Recently, several new exotic hypernuclear states were observed in the experimental investigations of high energy hadron-hadron collisions. These experimental observations on hypernuclei [6,7] have been collected by SKS, FINUDA Collaboration, DAΦNE machine, PANDA Experiment [8], KEK, and by a new facility in Japan, J-PARC [9,10] are expected that a higher aspect and new research fields will be given to hypernuclear physics like the research on exotic heavy hyperon systems properties and characteristics, which may be possible at new machines with higher energies. Therefore, the theoretical studies lead to greater awareness and interest in experimental interpretations. The discovery of hyperons (λ, Σ, Ω, Ξ) in high-energy reactions expands the scope of classical nuclear studies and broadens the scope of particle physics and nuclear astrophysics research. We address attempts to solve these difficulties in this Perspective, including tests with heavy-ion beams and the investigation of nuclear emulsions using cutting-edge technology. We provide an overview of current initiatives and experiments at different institutions across the globe, as well as future prospects.

THE EXOTIC MOLECULAR STATES

The Λ-hypernuclei are investigated as a dihadronic molecular system made up of either regular N or Λ-hypernuclei. The mass spectrum is calculated using principles from quantum scalar field theory and the projective unitary representation (PUR) approach. Fock explored variables describing and translating into four-dimensional momentum space for detailed analysis in the explanation of the hydrogen atom mass spectrum issue, which is similar to this approach [12]. We know that the Hamiltonian of a quantum harmonic oscillator reads in Eq. (1).

$$\hat{H} \cong \frac{\hat{p}^2}{2\mu} + \frac{\mu}{2}\omega_0^2 r^2 - E_{0j}(\mu) \tag{1}$$

In the pseudoharmonic potential between the two hadrons Λ-N, the Λ-hypermolecule Hamiltonian is considered to be in Eq. (2).

$$\hat{H}\Psi = E_j\Psi \quad \Rightarrow$$

$$\hat{H}\Psi = \left(M - \frac{1}{2}(\mu_1+\mu_2) - \frac{1}{2}\frac{{m_1}^2\mu_2 + {m_2}^2\mu_1}{\mu_1\mu_2}\right)\Psi$$

$$\hat{H} = \frac{\hat{p}^2}{2\mu} + V_0\left(\frac{r}{r_0} - \frac{r_0}{r}\right)^2 - E_{0j}(\mu) \tag{2}$$

where M is the mass of the hypermolecule bound state, m_1, m_2 is the rest mass of the lambda particle and N-nuclei, μ_1, μ_2 is the component mass or constituent mass of the Λ-hyperon, and N-nuclei: i.e. the mass of particles in the bound state that is different from the rest mass f particle, $\mu = \frac{\mu_1\mu_2}{\mu_{1+}\mu_2}$ is the reduced mass and it is the representation of a two-body system as a single-body one. When the motion (displacement, vibrational, rotational) of two bodies is only under mutual interactions, the inertial mass of the moving body with respect to the body at rest can be simplified to a reduced mass. $E_{0j}(\mu)$ is the dissociation energy between Λ-N or the absolute value of the binding energy B_Λ of the hypernuclear system [11,12], $E_{0j}(\mu)$ is the energy of the j-th excited state in the first approximation of PUR, r_0 is the equilibrium intermolecular Λ-N separation, in Eq. (2) it can be calculated by the empirical formula r $r_0 = 0.59 + 0.83A^{1/3}$, and it can be defined by experimental data.

We have not included the spin-orbit interactions in Eq. (1). Equation (1) reads: After variables representing $r = q^{2\rho}$ and translation into axillary $d = 2 + 2\rho + 4\rho\ell$ dimensional space in

$\rho = 0.5$, where Eq. (3):

$$\hat{H}_q = 4\mu\rho^2 q^{4\rho-2}\left(\frac{V_0}{{q_0}^2}q^2 + \frac{{q_0}^2}{V_0}q^{-2} - 2V_0 - E_{0j}(\mu)\right) + \frac{\hat{p}_q^2}{2} = \varepsilon_0(E,\mu) + H_0 + :H_I:, \quad (3)$$

where $r = q^{2\rho}$

$$\hat{p} = \frac{\hat{a}^- - \hat{a}^+}{2i}\sqrt{2\Omega}. \qquad \hat{p}^2 \cong \frac{d}{2}\Omega$$

$$\hat{q} = \frac{\hat{a}^- + \hat{a}^+}{\sqrt{2\Omega}}. \qquad \hat{q}^2 \cong \frac{d}{2\Omega}.$$

$\hat{p}_q^2$ is -N's relative momentum, Λ-N, $H_0 = \Omega(\hat{a}^+\hat{a}^-)$ is the free oscillator's energy : H_I: is the interaction Hamiltonian, $\varepsilon_0(E,\mu)$ is the ground state energy of the bound state in PUR, and is the highest variation estimate for the Hamiltonian's background energy. The entire Hamiltonian in the normal form, which does not include any perturbation order and terms with the order of $q^{2m},\ m < 1$, i.e., $:H_I: \approx 0$ in PUR [12], is used to describe ground and excited states in this article. Then, in the zeroth approximation of the PUR, the energy of the ground $j = 0$ and $j > 0$ excited states are calculated by reducing the expectation value of Hamiltonian, see Eq. (4):

$$\varepsilon_0(E,\mu) = \frac{\hat{p}_q^2}{2} + \frac{V_0}{q_0^2}\mu\hat{q}^2 - 2\mu V_0 - \mu E_{0j}(\mu) = 0 \Rightarrow$$

$$\varepsilon_0(E,\mu) = \Omega\frac{(1.5+\ell)}{2} + \frac{(1.5+\ell)}{\Omega}\mu\frac{V_0}{q_0^2} - 2\mu V_0 - -\mu E_{0j}(\mu) = 0 \quad (4)$$

$\varepsilon_0(E,\mu)$ is the free oscillator Hamiltonian or the exotic molecular bound state's minimal energy (i.e. the energy of the ground state or vacuum of the Hamiltonian). We may write $\frac{\partial\varepsilon_0(E,\mu)}{\partial\Omega} = 0$, and all quadratic terms are totally included in the free oscillator. As a result, the PUR 's energy eigenvalues and oscillator free frequency are as follows Eq. (5):

$$E_{0j}(\mu) = -2V + \Omega\frac{(1.5+\ell)}{2\mu} + \frac{(1.5+\ell)}{\Omega}\frac{V_0}{q_0^2} = -2V_0 + \frac{(1.5+\ell)\sqrt{2V_0}}{q_0}\mu^{-\frac{1}{2}}.$$

$$\Omega = \sqrt{\frac{2\mu V_0}{q_0^2}} = \mu\Omega_0 \quad (5)$$

and the ground state energy in the zeroth perturbation order j=0 in PUR [12] determines the pseudoharmonic energy spectrum, see Eq. (6).

$$E_{00}(\mu) = -2V_0 + \frac{1.5\sqrt{2V_0}}{q_0}\mu^{-\frac{1}{2}} \quad (6)$$

Then, in the zeroth approximation of PUR with nuclear core recoil effect, the mass spectrum of-hypernuclear bound state, see Eq. (7):

$$M = \mu_1 + \mu_2 + \mu\,\dot{E}_{0\ell}(\mu) + E_{0\ell}(\mu)$$

$${\mu_1}^2 = m_1^2 - 2\mu^2\dot{E}_{00}(\mu)$$

$${\mu_2}^2 = m_2^2 - 2\mu^2\dot{E}_{00}(\mu)$$

$$\dot{E}_{00}(\mu) = \frac{\partial E_{00}(\mu)}{\partial\mu} = -\frac{1}{\mu^{\frac{3}{2}}}\sqrt{2V_0}\frac{(1.5+\ell)}{2q_0} \quad (7)$$

Reference [12] gives the values V_0, q_0 for combinations of Λ-N, and parameter μ is the equation's root, see Eq. (8).

$$\mu^{-1} = \left(m_1^2 + \mu^{\frac{1}{2}}\sqrt{2V_0}\frac{(1.5+\ell)}{q_0}\right)^{-\frac{1}{2}} + + \left(m_2^2 + \mu^{1/2}\sqrt{2V_0}\frac{(1.5+\ell)}{q_0}\right)^{-\frac{1}{2}} \quad (8)$$

We calculated the mass spectrum and binding energy of Λ-hypernuclei without recoil effect $\ell = 0$, and the results are shown in Table 1.

$$M = m_{core} + \mu_\Lambda + \mu^{-\frac{1}{2}}\sqrt{2V_0}\frac{(1.5+\ell)}{2q_0} - 2V_0 \quad (9)$$

$$E_{bin} = \mu + \mu^{-\frac{1}{2}}\sqrt{2V_0}\frac{(1.5+\ell)}{2q_0} - 2V_0 \quad (10)$$

$$\mu_\Lambda = (m_\Lambda^2 + \mu^{1/2}\sqrt{2V_0}\frac{1.5}{q_0})^{1/2} \quad (11)$$

$$\Omega_0 = (\frac{2V_0}{q_0^2})^{1/2}\mu^{-1/2} \quad (12)$$

Then, without the recoil effect of the nuclear core ($m_2 = m_{core} = \infty$), compute numerical values of parameters of the Λ-hypernuclei in the ground and excited states. When we compare the findings of Table I, we can observe that the ground state results produced numerically and analytically in PUR are quite similar.

Table 1. Calculated mass and energy spectrum, Λ-hyperon component mass ($A < 10$), Λ-hypernuclei binding energies, and oscillator frequency in ground states (in MeV). References [13,14,17,18] provide theoretical and experimental evidence.

	M	μ_Λ	E_{00}	E_{bin}	M_{exp}	M_{theory}
${}^{8}_{\Lambda}He$	7675.141	1115.720	14.239	7.443	7653.2	-
${}^{8}_{\Lambda}Li$	7674.283	1115.721	13.622	7.183	7642.52	7663.42
${}^{8}_{\Lambda}Be$	7673.151	1115.722	13.606	7.225	7642.86	-
${}^{9}_{\Lambda}Li$	8610.527	1115.723	16.980	8.904	8578.69	-
${}^{9}_{\Lambda}Be$	8612.957	1115.719	15.349	7.092	8563.69	-

For example, our findings for the masses of hypernuclei like ${}^{8}_{\Lambda}He$, ${}^{8}_{\Lambda}Li$, and ${}^{9}_{\Lambda}Be$ are 7675.141, 7674.283, and 8612.957 MeV, respectively, while their experimental values in [14] are 7653.2, 7642.52, and 8563.69 MeV. We also looked into the binding energy of ${}^{8}_{\Lambda}He$, ${}^{8}_{\Lambda}Li$, ${}^{8}_{\Lambda}Be$, ${}^{9}_{\Lambda}Li$, and ${}^{9}_{\Lambda}Be$ made a graph of the binding energy vs mass number A, as shown in Fig 1. Our findings were compared to theoretical and experimental studies [16]. It should be noted that the experimental and theoretical findings show a high level of agreement.

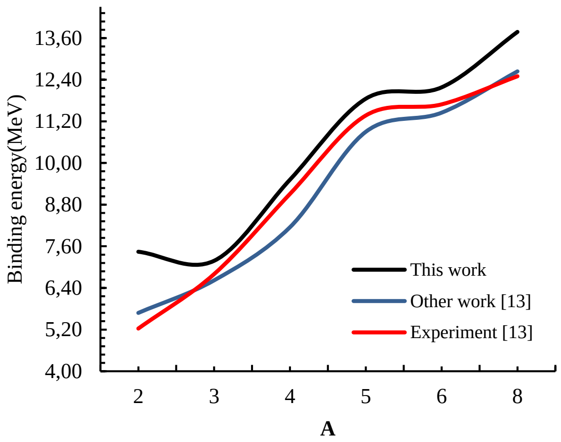

Fig. 1. The binding energy of hypernuclei versus the mass number.

RESULTS AND DISCUSSION

By treating the Λ-N states as dihadronic molecules, I was able to calculate their masses. The hadronic interaction has been assumed, and an analytic approach for calculating the mass spectrum and component masses is proposed based on the asymptotic behavior of the correlation functions of the related field currents. While the hadronic binding energy was derived from experimental data, the pseudoharmonic and dihadronic molecule binding energies were calculated using several methods, including PUR. For determining the mass spectrum of Λ-N states, the PUR approach is proposed. This approach has the benefit of being able to include a wide range of unusual bound states.

As a result, I've shown that the masses of Λ-hypernuclei can be efficiently estimated using the yields of PUR and QFT techniques with the $\ell = 0$ and $\ell \neq 0$ parameters. Table 1 compares and identifies the findings with some of the experimentally known unusual hypernuclear atoms. This approach may also be used to detect the binding energies and mass spectrum of exotic multi hyperon nuclei, which have been difficult to quantify in earlier hyper physics studies. Such unusual multi hyperon nuclei, I think, will be attainable in the next generation of intermediate and high-energy ion accelerators. Many unusual dihadronic states $J^{PC} = 0^{++}$ might be found at these energies at various sectors. Many of these hypernuclei may be linked to exotic hadronic states that have been seen experimentally. Only a few unusual Λ-N states are provided in Table1 among the various combinations of hyperon-hadron nuclei and dihadronic molecular states that may be explored.

Despite the fact that the meaning of ${}^{A}_{\Lambda}Z$ is still debated, I identify it as the Λ-${}^{A-1}_{Z}Z$ molecular state. Many more anticipated dihadronic states, such as ${}^{A}_{YY}Z$, might be discovered experimentally. Hypernuclei's theoretical masses must be calculated. They are compared to other theoretical works in certain cases. An extensive study is underway to determine the precise nature of unusual hypernuclei correlation. As a result, hypernuclei are described in the present study as a composite complex quark state. The findings are promising and might help us better understand the underlying ingredient of new exotic matter, hypernuclei and the hyperon issue in neutron stars, long-lived hyper weird multiquark droplets, and strange quark matter. Thus, understanding the structure of exotic atoms and exotic heavy hypernuclei, unusual compact stars, and exotic Λ-N interactions requires a thorough grasp of the features of Λ-hypernuclei. Then, using Eqs. (9-12), we provide our numerical findings for the *n=1* state binding energies, hypernuclei mass, and component mass of particle in heavy Λ-hypernuclei (12). As can be observed from Eq. (7), there is a parameter μ that can be derived using theoretical component masses (μ_1, μ_2) for hypernuclei rest mass. We utilize the QFT and PUR ground state binding energies of Λ-hypernuclei to calculate these values. We estimated our theoretical for the ground state of heavy Λ-hypernuclei using the ideal reduced mass parameter and the potential parameters V_0 and q_0, and compared it to theoretical and experimental evidence from [14-18].

The validity of the mass parameters derived is ensured by a satisfactory match between theoretical and experimental data. We used a two-body simple model Λ-hypernuclear as an exotic molecule bound system resembling a dihadronic molecule: hyperon-core (N) to determine the pure oscillator frequency and binding energy of Λ-particle in hypernucleus. This is a useful model for dealing with the complexities of multiparticle exotic systems calculations. Table 1 shows that raising the mass number of the Λ-hypernucleus ${}^{8}_{\Lambda}Be$,${}^{9}_{\Lambda}Be$ increases the value of the component mass of the Λ-particle and the value of the energy eigenvalue E_{00} of the Λ-hypernucleus.

Additionally, raising the proton number of the Λ-hypernucleus (${}^{8}_{\Lambda}He$,${}^{8}_{\Lambda}Li$,${}^{8}_{\Lambda}Be$) raises the value of the component mass of the Λ-hyperon. It's worth noting that the charge dependency of the hyperon-core (Λ-N) interaction, i.e. the proton-hyperon bound state contact is greater with more protons than with more neutrons. Despite the electrostatic repulsion of protons, which causes the ${}^{8}_{\Lambda}Li$ reduction energy eigenvalue to be larger than the ${}^{8}_{\Lambda}He$, the ${}^{8}_{\Lambda}Li$ is higher than the ${}^{8}_{\Lambda}He$.

As we know ${}^{A}_{\Lambda}He$ the criteria for picking these heavy exotic dihadronic compounds is their immense relevance in hyper nuclear physics, weird stars, and neutron star related fields. White dwarfs, neutron stars, and heavy-ion collision experiments all require He and ${}^{A}_{\Lambda}Li$, which are significant dihadronic molecules involved in many nuclear processes.

CONCLUSION

Recent theoretical investigations on heavy hyperon hypernuclei are presented. In the PUR framework of the potential model, we address the usage of the Wick ordering approach to calculate mass spectrum and conduct a complete research of heavy Λ-hypernuclei systems. Exotic hypernuclei characteristics have been studied using the prospective model parameters and masses of the Λ-hypernuclei derived from the corresponding hyperon-core mass predictions. Exotic hadronic states are used to represent some of my bound state predictions. In the future generation facilities, such as J-PARC, MAMI, JLab, and FAIR, some new exotic hypernuclei-bound states might be found experimentally very soon. The limits on the V 0.q 0 parameters have been presented for the one-dimensional Schrödinger equation with a pseudoharmonic potential after analysis. Under the effect of PUR, the issue is solved in axillary d-dimensional space, and the bound state energy solutions of exotic Λ-hypernuclei are obtained. It is possible to obtain relativistic energy levels, mass spectrum, and constituent mass. In a related application, I calculated the pure oscillator frequency of a few exotic Λ-hypernuclei as dihadronic molecules and created an interest in these exotic molecules in order to further explore them in future harmonic, molecular, Yukawa, and Coulombic potentials research.

The Y-Y connection may be accessible by double Λ-hypernuclei, similar to how hypernuclei confine the Y-N interaction. Due to a lack of evidence, the potential is now underappreciated. There have only been a few double Λ-hypernuclei identified. The moderate synthesis rate of light double Λ-hypernuclei, such as ${}^{5}_{\Lambda\Lambda}H$, ${}^{5}_{\Lambda\Lambda}He$, and others, according to thermal model predictions, provides heavy ion investigations finding potential. The finding of these bound states, as well as the determination of the binding energy, is expected to have a significant influence on our knowledge of the Y-Y interaction, which has fundamental implications for nuclear and astrophysics. The accurate solution of the radial Schrödinger equation, which is already done in this study, is one of the most important phases in correlator computation.

This study employs systematic methods, and it is, in many ways, one of the most concrete works in this field. The extended pseudoharmonic potential, in particular, may be a significant potential, and it merits special attention in many disciplines of physics, including hadronic, nuclear, and atomic physics. As a result, looking for an analytical solution to the modified radial Schrödinger equation for the sum of the pseudoharmonic, Cornell, inverse quadratic-, and harmonic-type potentials within the framework of ordinary quantum mechanics and quantum field theory could provide useful information on elementary particle physics and quantum chromodynamics, as well as open up new avenues for research. Because of the precise and more general character of the findings, we may infer that the theoretical outcomes of this work are predicted to open up new opportunities for pure theoretical and experimental physicists.

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AUTHOR CONTRIBUTION

I wrote the manuscript and determined data for Table 1 and Fig. 1, and conducted all theoretical and analytical analyses.

REFERENCES

1. N. Buyukcizmeci, A. S. Botvina, R. Ogul *et al.*, Eur. Phys. J. A **56** (2020) 210.
2. C. Dover and A. Gal Ann. Phys. **146** (1983) 309.
3. A. Jahanshir, Arab J. Nucl. Sci. Appl. **55** (2022) 127.
4. A. Jahanshir and M. R. Soltani, Karbala Int. J. Mod. Sci., **7** (2021) 437.
5. A. Gal, EPJ Web Conf. **259** (2022) 08002.
6. Y. H. Leung, EPJ Web Conf. **259** (2022) 08001.
7. T. R. Saito, W. Dou, V. Drozd *et al.*, Nat. Rev. Phys. **3** (2021) 803.
8. A. Gal, E. V. Hungerford and D. J. Millener, Rev. Mod. Phys. **88** (2016) 035004.
9. Anonymous, The first determination of the Xi hypernuclear mass - New insights toward understanding of the origin of nuclei and structure of neutron stars. https://j-parc.jp/c/en/press-release/2021/03/02000662.html. Retrieved in May (2021).
10. T. Gogami, S. Kanatsuki, T. Nagae *et al.*, J. Phys. Conf. Ser. **1643** (2020) 12133.
11. T. Nagae, Few-Body Syst. **62** (2021) 1.
12. M. Dineykhan, G. V. Efimov, G. Ganbold *et al.*, Oscillator Representation in Quantum Physics, 2nd ed., Springer-Verlag, Berlin (1995) 210.
13. N. N. Kolesnikov and S. A. Kalachev, Phys. Part. Nucl. Lett. **3** (2006) 341.
14. J. Pniewski, Acta Phys. Pol. B **129** (1971) 129.
15. S. Pal, R. Ghosh, B. Chakrabarti *et al.*, Phys. Scr. **95** (2020) 045301.
16. S. Pal, R. Ghosh, B. Chakrabarti *et al.*, Eur. Phys. J. Plus. **132 (**2017) 262.
17. P. A. Zyla, The Review of Particle Physics Archived 2020 Edition. https://pdg.lbl.gov/rpp-archive/index.html 083C0. Retrieved in May (2021).
18. A. Armat and S. M. M. Nejade, Int. J. Mod. Phys. E **28** (2019) 1950011.