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The Role of Neutron Absorbers in Soliton Wave Creation Using Heavy Water as a Diffusive Medium

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ABSTRACT

One of the simplest nuclear fission reactor designs is the soliton reactor. In these reactors, neutrons reduce the toxicity of fissile materials in a manner that allows new vital areas appear successively. Therefore, the spatial dependence of the neutron flux, specific power density, and associated particle density exhibit wave phenomena of solitons and emerge from the solution of nonlinear partial differential equations, preserving their shape during propagation. The velocity of the burnup Soliton Wave (SW) is related to the density of the initial Nuclear Fuel (NF) in each Neutron Absorber (NA) in the medium. These nonlinear waves can be described by equations describing the atomic flux and density in terms of time and space in the medium. The soliton wave can also be observed in advanced nuclear power systems. Burnup SWs in a propagation medium can be analyzed using the spatial coordinates and position of the NA in a propagation region. The aim of this work is to investigate the burnup SW characteristics by selecting various isotopic neutron absorbers in the slab reactor core. Our computational findings show that the SW burning rate is affected by increasing the diffusion coefficient. However, both the diffusion length and the Length of Transient (LOT) increase with increasing the diffusion coefficient. Interestingly, the ratio of LOT to diffusion length remains constant. Furthermore, while increasing the diffusion coefficient leads to a higher Transient of Time (TOT), the ratio between TOT and characteristic time remains constant.

INTRODUCTION

In the 19th century, Russell observed waves on the water surface and referred to them as Soliton Waves (SWs) [1]. These waves continue to travel for extended periods while maintaining their original characteristics. They remains unchanged in shape and speed along their path. SWs are localized, highly stable waves that preserve their nature over time. At the end of the 19th century, Korteweg and De Vries introduced the KdV Eq. for soliton waves. This equation contains nonlinear terms, which describe the movement of SWs with small amplitude in diffusive media [2]. This equation provides a comprehensive model for studying nonlinear waves in dispersive media [3–6]. The easiest form of KdV Eq. is [7]: $u_t + auu_x + u_{xxx} = 0$, where u_t represents the evolution of wave propagation with time in the x dimension. In addition, the KdV Eq. contains two important points: 1) It shows the nonlinearity, which is represented by uu_x and estimates SW slope, and 2) It shows the linear dispersion, which is represented by u_{xxx}, which estimates the propagation of SW. Nonlinearity localizes the wave, but dispersion causes it to spread. Studies have shown that in several nonlinear environments, for example in shallow water, the wave packet broadening caused by scattering can be balanced by the effects of

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narrowing caused by the nonlinear environment. The balance describes the formulation of solitons. The stability of SWs results from the balance between nonlinear and dispersive effects. This equation yields soliton resonators that characterize SWs with particle identities that decay at long distances.

In 1965, a large SW nonlinear interaction that overtook a smaller SW was numerically studied by Zabusky & Kruskal [3]. Their research findings revealed that SWs obey the KdV equ. under nonlinear interaction. The fact that SWs retain their identity and behave like particles has motivated researchers to explore them further [3]. The interaction between two solitons is governed by the fact that their shape, speed, and pulse characteristics remain unchanged, and the collision is elastic. In 1995, research on burnup wave propagation in pure Neutron-Absorbing Media (NAM) was reported by Seifritz [4]. This problem was analyzed theoretically, and it was found that the reaction rate of neutrons propagates in a diffusive medium like an SW. Then, Van Dam investigated the effect of SW on burnup wave propagation from different aspects [5–7]. In analyzing neutron behavior in a nuclear reactor, one of the most important parameters is determining the NF in each area of the reactor core. Accurately evaluatiing this NF is essential for determining the the reactor's power distribution and other parameters critical to its safe operation [8]. The aim of this work was to investigate on the burnup SW characteristics by selecting various isotopic Neutron Absorbers (NAs) such as Boron (^{10}B), Cadmium (^{113}Cd), Samarium (^{149}Sm), Europium (^{151}Eu), Hafnium (^{177}Hf) and Gadolinium (^{157}Gd) with heavy water as a diffusive media, using the net input neutron current to be incident normal on the slab boundary [9,10].

THEORY/CALCULATION

The KdV Eq. includes the dynamics of hydrodynamic and lattice fields. This equation is an example of a wide range of SW equations. Here, to examine some of the physical properties of solitons, we examine the equation of Sine Gordon, Eq. (1):

$$\varphi_{tt} - \varphi_{xx} + sin\varphi = 0 \tag{1}$$

There are several different solutions for this equation, and since x belongs to the range of -∞ to +∞, the solution belonging to the range of 0 to 2π is a soliton solution. The solution of anti-soliton belongs to the range 0 to 2π. In 1939, the Sine Gordon Eq. was proposed as a model for crystal defects [11] also subsequently, in the 1960s, elementary particles were analyzed by the Sine Gordon Eq. [12]. They investigated soliton and anti-soliton collisions and confirmed the stability of particles [13]. In addition, scientists introduced the Q quantity as follows Eq. (2):

$$Q = \frac{1}{2\pi}\{\varphi(\infty, t) - \varphi(-\infty, t)\} \tag{2}$$

Q must be a constant under the boundary conditions governing the selected system. In the 1960s, interesting phenomena in the field of SWs behavior were introduced by researchers in the field of nonlinear optics [14]. Laser light that is emitted from 2-level lasers obeys the Sine Gordon equation [15]. One of the other applications of the Sine Gordon equ. is in the Josephson junction with magnetic flux diffusion. The Josephson junction is made of two superconductors and an insulator. We present ŋ_1 and ŋ_2 as a phase of cooper pair wave functions in superconducting plates. According to the Josephson current $j = j_0 sin\text{ŋ}$ due to the $\text{ŋ} = \text{ŋ}_1 - \text{ŋ}_2$ the motion of ŋ is explained by Sine Gordon equation. In general, The Sine Gordon Eq. has many applications in various branches of technology. SWs have been widely used in various fields of physics such as plasma, nonlinear optics, condensed matter, low temperature, particles, biophysics, hydrodynamics, nuclear fission reactor, and astrophysics [16–22].

METHODS

Properties of selected neutron absorbers

Burnable absorbers, also known as burnable neutron poisons, are materials placed in the core of a fission reactor and include fertile nuclei with a high absorption cross-section (σa). These absorbers capture neutrons, significantly reducing their number throughout the reactor's operational cycle. As the inventory of burnable absorbers decreases, their impact on reactivity diminishes. Burnable absorbers play a crucial role in controlling reactivity over long-term fuel cycles [23–27]. In practice, burnable absorbents are often used to reduce the maximum reactor power. burnable absorber burns effectively at a rate equal to the fuel rate, so the resultant reactivity does not change with time. In the operating fuel cycle, if burnable absorber burns with a high rate, the positive feedback oscillations can exceed from limits of fission reactor, while if the burnable absorber burns at a low rate, the remaining burnable absorber will result in negative feedback oscillations.

Fuel rods containing gadolinium are applied to control the reaction rate of boiling water reactors, while boron solutions are used to control the reaction rate in the pressurized water reactors.

Since neutron absorption cross-section in burnable poisons is usually a function of energy, therefore burnable absorbers are related to temperature variations. The properties of 6 different burnable absorber isotopes are briefly presented below. I) Boron (^{10}B): its neutron absorption cross section reduces exponentially with neutron energy. II) Gadolinium (^{155}Gd and ^{157}Gd): These two isotopes of gadolinium are burnable absorbers with resonances energy higher than 1eV. Depending on the spectrum of the reactor, these two isotopes will have an increase in reaction rate as a function of temperature. Some gadolinium isotopes are stable, and some of them are strong neutron absorbers. Also, ^{157}Gd has a very high thermal absorption cross section. III) Europium (^{167}Er): This isotope has a large thermal resonance at 1eV which leads to an increased absorption and thus a negative reaction rate in terms of temperature. Note that in fast reactors, boron and gadolinium are two common burnable absorbers. One of the main challenges with this isotopes is their separation, which is costly. IV) Hafnium (Hf): Although hafnium-based materials are not very common, they are good burnable absorber candidates due to their ability to absorb neutrons with more than thermal energy. If a neutron is absorbed by hafnium, another isotope from the hafnium element is formed, which is a neutron absorber. If a layer of hafnium dioxide is created on the surface of the Hf metal, it will show a good corrosion resistance against hot water. In a long-term fuel cycle, V) Cadmium: Cd-based burnable absorbers will show better neutronic behavior. Cd is one of the components of control rods containing Ag-In-Cd in fission reactors [28–30]. Cadmium-based burnable absorbers are made of cadmium wires or cadmium oxide pellets [31]. However, the use of Cd-based burnable absorbers in fission reactors will incur significant costs for their production and safe storage due to health challenges.
VI) Samarium: The samarium element has six stable isotopes. Among them, the ^{149}Sm isotope constitutes approximately 14% of natural samarium, which is a neutron absorber with a high cross-section. In this paper, we focus on the above burnable absorbers with their properties in Table 1, where $\sigma_{a,th}$ and $\sigma_{a,fast}$ are the thermal and fast cross sections at neutron energy 0.025 eV and 200 keV, respectively.

Table 1. Properties of selected burnable absorbers.

Burnable absorbers	$T_{1/2}$	Abundance%	$\sigma_{a,th}$ (b)	$\sigma_{a,fast}$ (b)
Boron (^{10}B)	stable	19.9	9.38	1.97
Cadmium (^{113}Cd)	$8.0 \times 10^{15} y$	12.2	38	0/31
Samarium (^{149}Sm)	stable	13.8	40150	0.70
Europium (^{151}Eu)	$5.0 \times 10^{18} y$	47.8	9185	0.30
Hafnium (^{177}Hf)	stable	18.5	375	0.55

Suggested model

Neutron diffusion theory provides the key to calculate NF in the nuclear fission reactor. One of the important parameters in the organization of a nuclear fission reactor is neutron distribution. Neutrons encounter different interactions while moving in the system. Today, diffusion theory is widely used in the core of pressurized and boiling water reactors. This theory describes the NF quite logically. The theoretical geometry used for this work is a 1D semi-infinite slab to obtain the soliton transition and the stationary state in the burning diffusive medium. The selection of this model presents the role of new quantities used in the development of SWs in a nuclear fission reactor diffusive media. If a speed increase is required, the NA density must be diluted by introducing neutron moderators into the reactor core. The distribution of NF and burning properties of a diffusive medium with a NA will be estimated through the neutron diffusion equ. and the burning equ. The neutron diffusion Eq. (3) is [6]:

$$D\frac{\partial^2\varphi(x,t)}{\partial x^2} - \sigma N(x,t)\varphi(x,t) = \frac{1}{v}\frac{\partial\varphi(x,t)}{\partial t} \tag{3}$$

Where $\varphi(x,t)$, $N(x,t)$, σ, D and v are NF, density of NA, absorption cross-section of NA, diffusion coefficient, and neutron speed, respectively. In this work, we have chosen the geometry presented in Fig. 1.

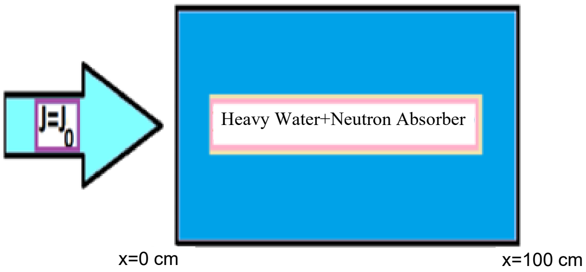


Fig. 1. The slab geometry of neutron diffusive and absorbing media under irradiation of neutron current.

Table 2. Characteristics of selected NAS in diffusive medium.

Burnable absorbers	Cross-section (b) $\times 10^3$	Burnup wave Velocity (cm/day)	Diffusion length (cm)	Number density (neutron/cm^3) $\times 10^{19}$
Boron (^{10}B)	3.84	0.83	4.6	1.04
Cadmium (^{113}Cd)	20.76	4.55	4.6	0.19
Samarium (^{149}Sm)	41	8.80	4.6	0.098
Europium (^{151}Eu)	9.18	2.0	4.6	0.44
Hafnium (^{177}Hf)	0.37	0.09	4.6	11

Since the change of NF with time is very small while the neutron velocity is very high, we can eliminate the time derivative expression, so the above equation becomes: Considering that the change of the NF with time is very low while the speed of the neutron is very high, therefore we can omit the time derivative expression so the above Eq. (4) becomes as follows:

$$D\frac{\partial^2\varphi(x,t)}{\partial x^2} - \sigma N(x,t)\varphi(x,t) = 0 \qquad (4)$$

Also, the NA burnup Eq. (5) can be written as:

$$\frac{\partial N(x,t)}{\partial t} = -\sigma N(x,t)\varphi(x,t) \qquad (5)$$

The boundary and initial conditions for selected geometry are: $N(x,t) = N_0$, $\varphi(100,\mathrm{t}) = \varphi_0$ and $J(0,t) = J_0$, here J is known as the current density of neutron. We assume that NF is canceled on the boundary. Another important parameter is reaction rate, which is given by Eq. (6):

$$R(x,t) = \sigma N(x,t)\varphi(x,t) \qquad (6)$$

We can solve each of Eqs. (4) and (5) using the asymptotic solution. Selecting the suitable boundary conditions, we will have the following asymptotic solutions based on Eqs. (7) and (8):

$$N_{as}(x,t) = \frac{N_0}{\left(1+\exp\left(-\frac{x-vt}{l_0}\right)\right)^{3.5}} \qquad (7)$$

$$\varphi_{as}(x,t) = \frac{v\sqrt{2}}{\sigma l_0}\left(\frac{N}{N_0} - 1 - \ln\left(\frac{N}{N_0}\right)\right)^{\frac{1}{2}} \qquad (8)$$

and for the asymptotic reaction rate density can be written as Eq. (9):

$$R_{as}(x,t) = \frac{v\sqrt{2}}{l_0}N_{as}\left(\frac{N_{as}}{N_0} - 1 - \ln\left(\frac{N_{as}}{N_0}\right)\right)^{\frac{1}{2}} \qquad (9)$$

Diffusion length is calculated by: $l_0 = \sqrt{\frac{D}{N_0\sigma}}$. The wave speed is obtained from the relationship $v = \frac{\varphi_0}{N_0}$, this result can be obtained by considering the point that the number of fuel absorbing nuclei per unit of time is equal to the number of neutrons entering the medium divided by time. The correlation is shown by Eq. (10):

$$\begin{aligned}\varphi_0 &= -\frac{d}{dt}\int_0^\infty N_{as}\,(x-vt)dx\\ &= -\int_0^\infty \frac{d}{dt}N_{as}(x-vt)dx\\ &= \int_0^\infty v\frac{d}{dx}N_{as}(x-vt)dx\\ &= vN_0 \qquad (10)\end{aligned}$$

Now, by using the asymptotic solutions, we can get exact solutions for the solitary waves resulting from Eqs. (4) and (5) which are expressed in Eqs. (11) – (13):

$$\varphi(x,t) = \varphi_0\left(\frac{1+\exp(-\sigma N_0 vt)}{1+\exp(\sigma N_0(x-vt))}\right) \qquad (11)$$

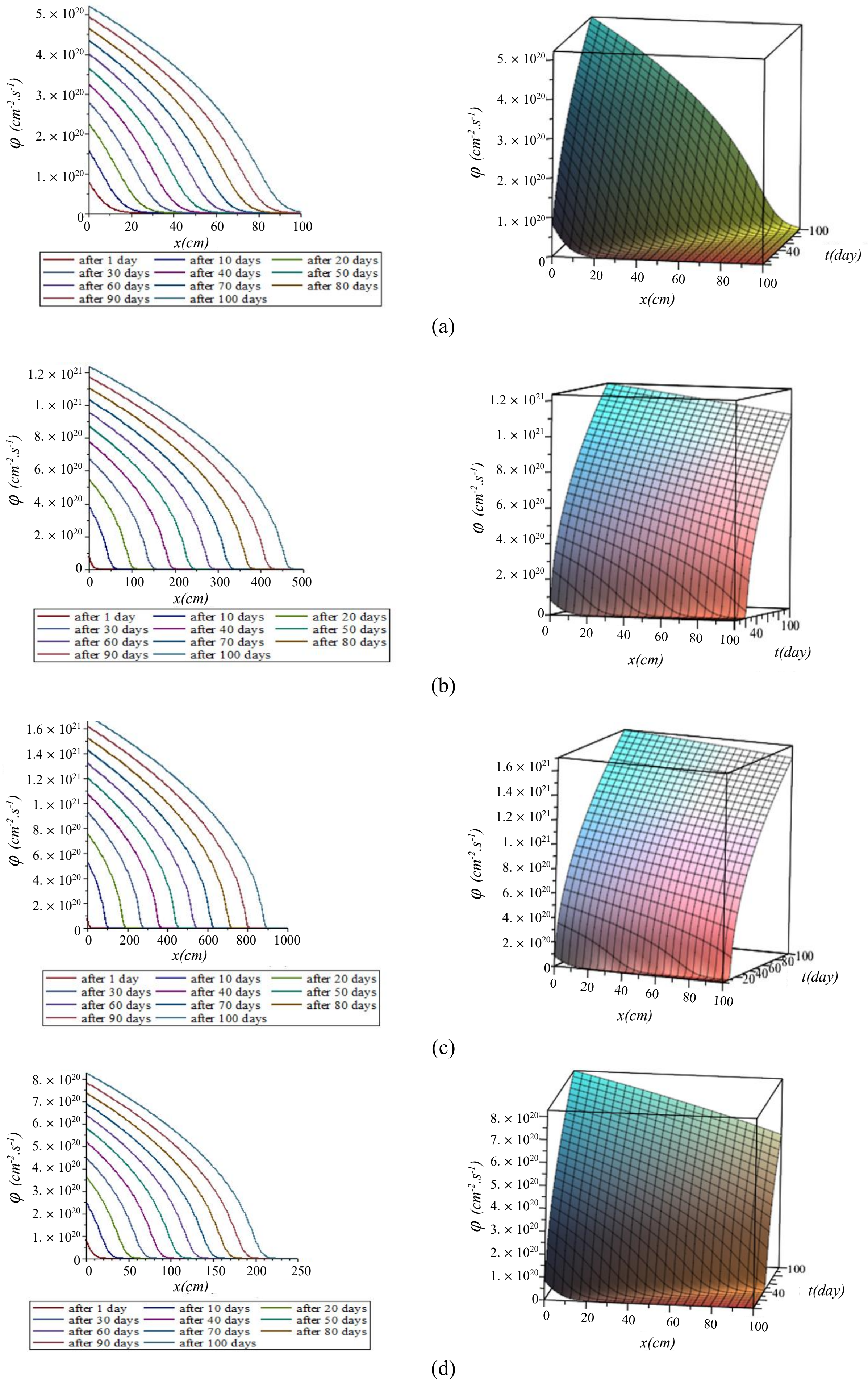
$$N(x,t) = N_0\left(\frac{1+\exp(\sigma N_0 x)}{\exp(\sigma N_0 vt)+\exp(\sigma N_0 x)}\right) \qquad (12)$$

$$R(x,t) = \sigma\, N(x,t)\,\varphi(x,t) \qquad (13)$$

In this work, we study on the nuclear burnup of SW characteristics for selecting various isotopic NAs such as Boron (^{10}B), Cadmium (^{113}Cd), Samarium (^{149}Sm), Europium (^{151}Eu), Hafnium (^{177}Hf) and Gadolinium (^{157}Gd). We assume that the net input neutron current is 10^{14} cm^{-2} s^{-1} and to be irradiated normal to slab edge at x = 0 (see Fig.1). In Table 2 the characteristics of selected NAS in diffusive medium are given.

RESULTS AND DISCUSSION

Using our suggested model, we plotted the two and three-dimensional NF, reaction rate and nuclide density diagrams versus time and position in D_2O diffusive medium for each of the selected NAs, respectively (See Figs. 2-4).

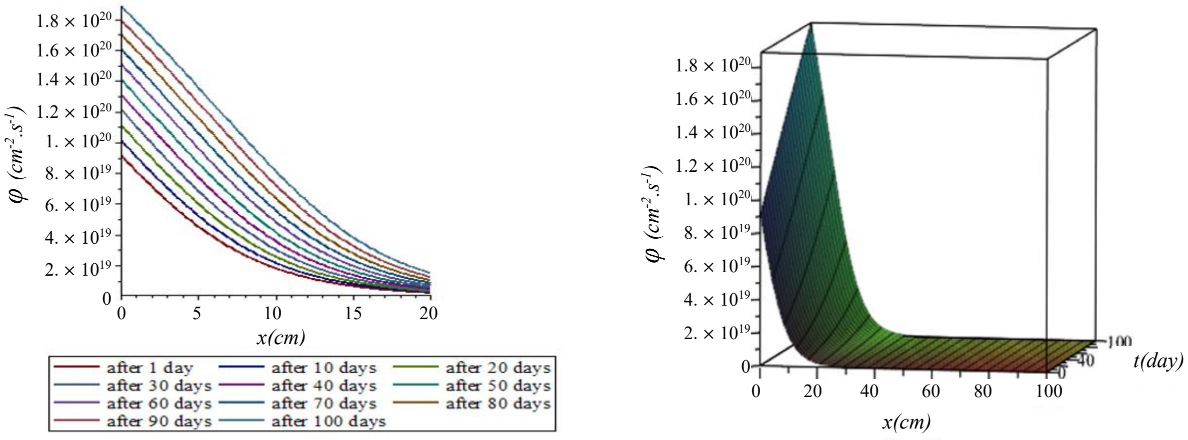

φ (cm⁻².s⁻¹)
x(cm)
t(day)
5. × 10²⁰
4. × 10²⁰
3. × 10²⁰
2. × 10²⁰
1. × 10²⁰
1.2 × 10²¹
1. × 10²¹
8. × 10²⁰
6. × 10²⁰
1.6 × 10²¹
1.4 × 10²¹
7. × 10²⁰
Φ (cm⁻².s⁻¹)
after 1 day
after 10 days
after 20 days
after 30 days
after 40 days
after 50 days
after 60 days
after 70 days
after 80 days
after 90 days
after 100 days

(a)

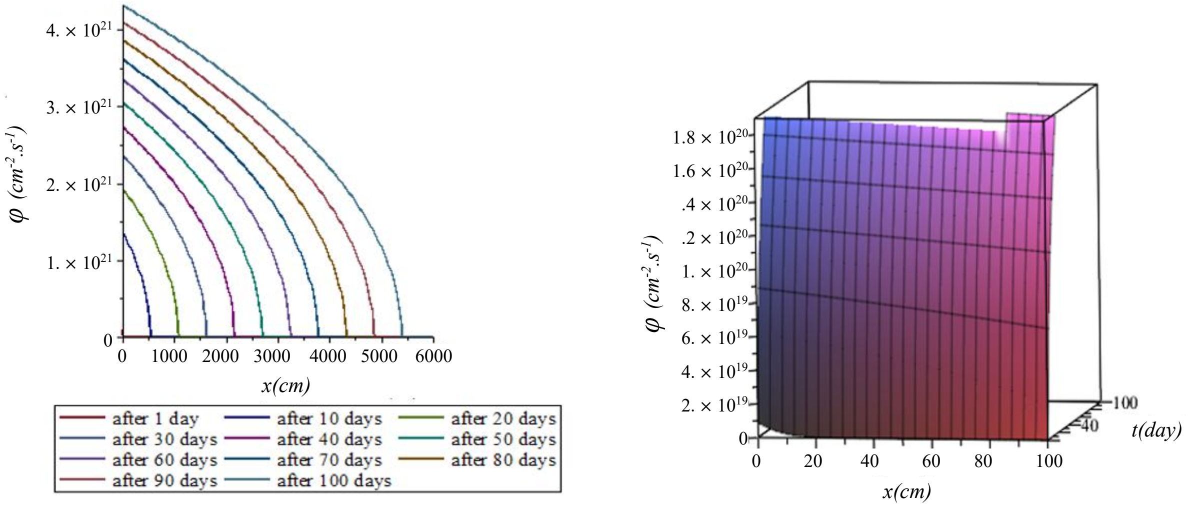
(b)

(c)

(d)

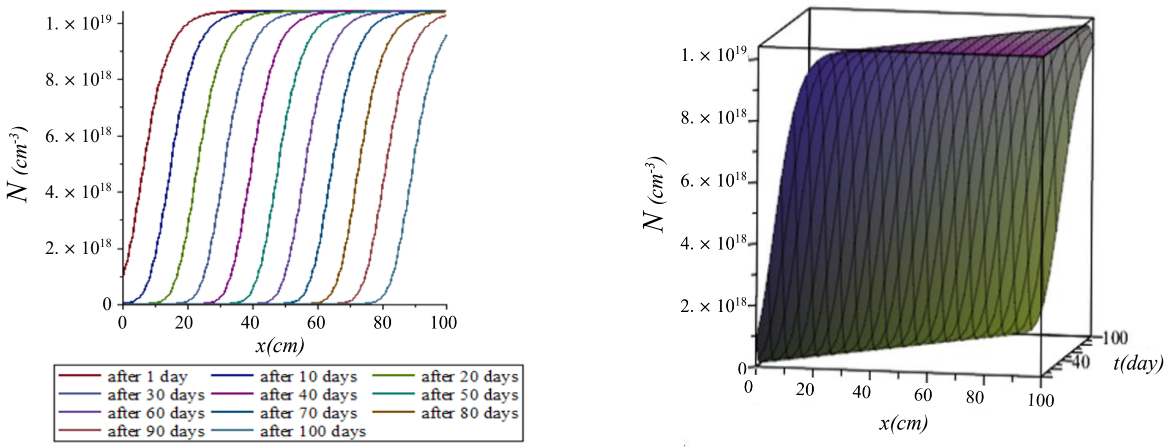


(e)

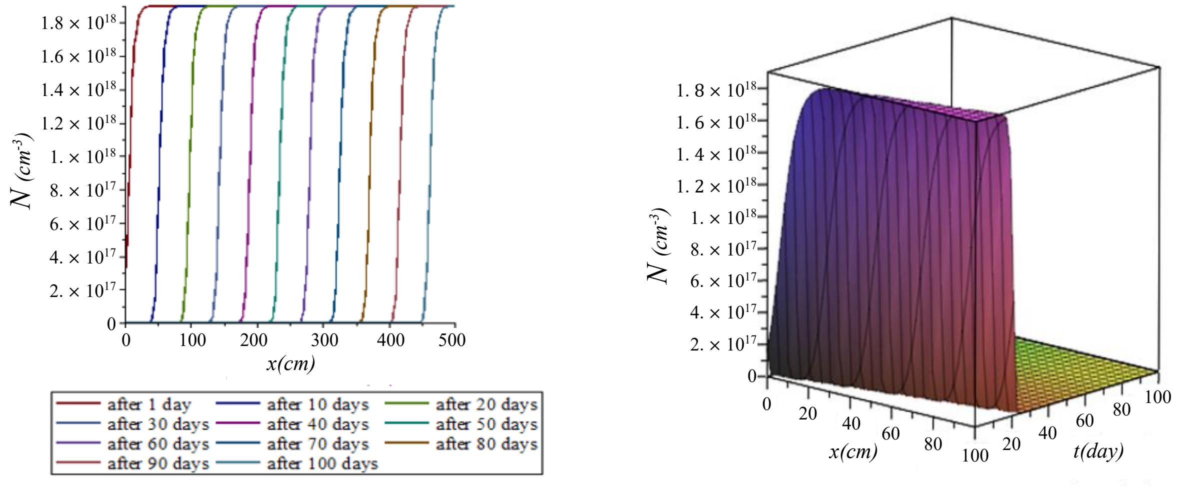


(f)

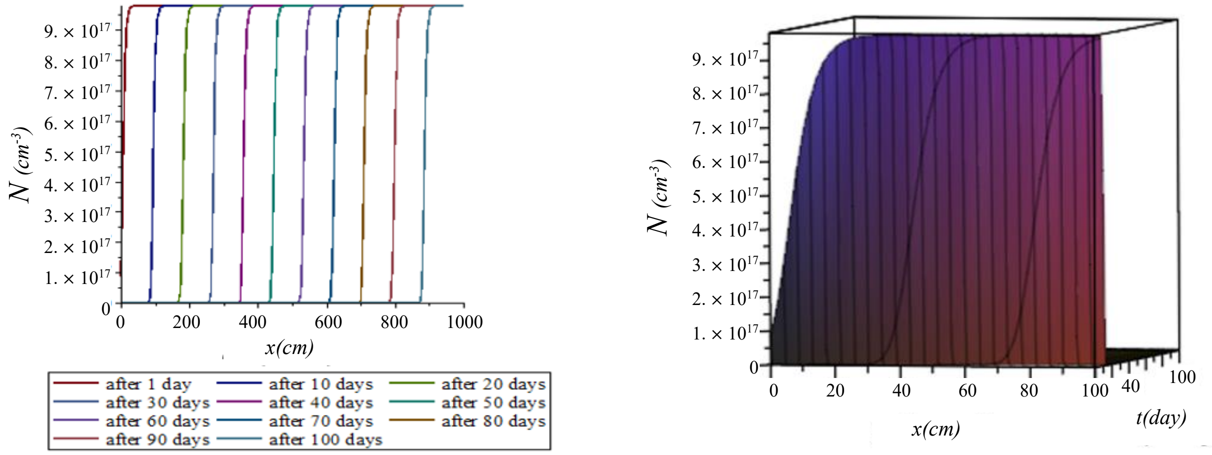
Fig. 2. Two and three-dimentional (2D and 3D) variations of φ for different neutron absorbing media: a) Boron (^{10}B), b) Cadmium (^{113}Cd), c) Samarium (^{149}Sm), d) Europium (^{151}Eu), e) Hafnium (^{177}Hf), and f) Gadolinium (^{157}Gd) as a function of position and time.



(a)


N (cm^{-3})
1.8 × 10^{18}
1.6 × 10^{18}
1.4 × 10^{18}
1.2 × 10^{18}
1. × 10^{18}
8. × 10^{17}
6. × 10^{17}
4. × 10^{17}
2. × 10^{17}
0
0
100
200
300
400
500
x(cm)
after 1 day
after 10 days
after 20 days
after 30 days
after 40 days
after 50 days
after 60 days
after 70 days
after 80 days
after 90 days
after 100 days
x(cm)
t(day)
0
20
40
60
80
100

(b)


N (cm^{-3})
9. × 10^{17}
8. × 10^{17}
7. × 10^{17}
6. × 10^{17}
5. × 10^{17}
4. × 10^{17}
3. × 10^{17}
2. × 10^{17}
1. × 10^{17}
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0
200
400
600
800
1000
x(cm)
after 1 day
after 10 days
after 20 days
after 30 days
after 40 days
after 50 days
after 60 days
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after 80 days
after 90 days
after 100 days
x(cm)
t(day)
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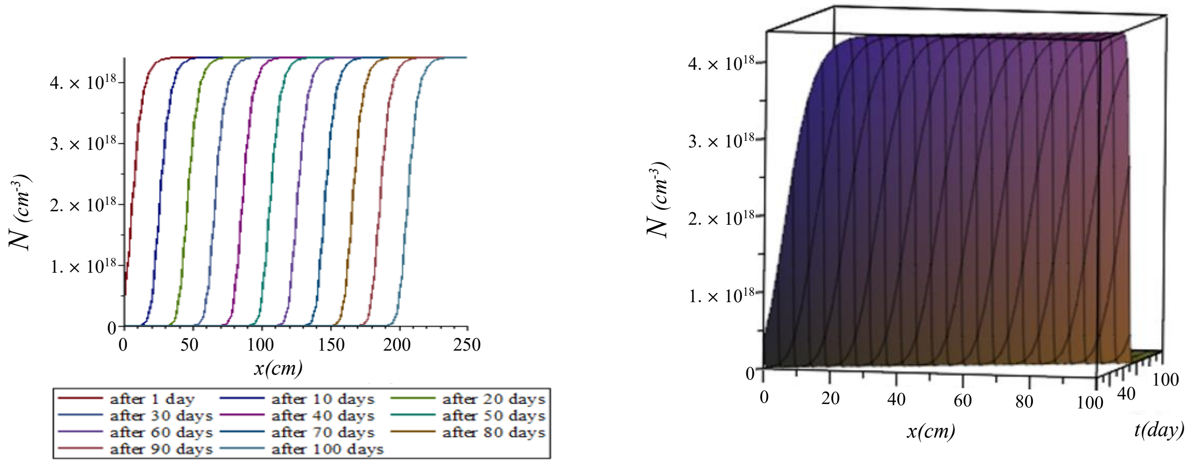
80
100
40

(c)


N (cm^{-3})
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3. × 10^{18}
2. × 10^{18}
1. × 10^{18}
0
0
50
100
150
200
250
x(cm)
after 1 day
after 10 days
after 20 days
after 30 days
after 40 days
after 50 days
after 60 days
after 70 days
after 80 days
after 90 days
after 100 days
x(cm)
t(day)
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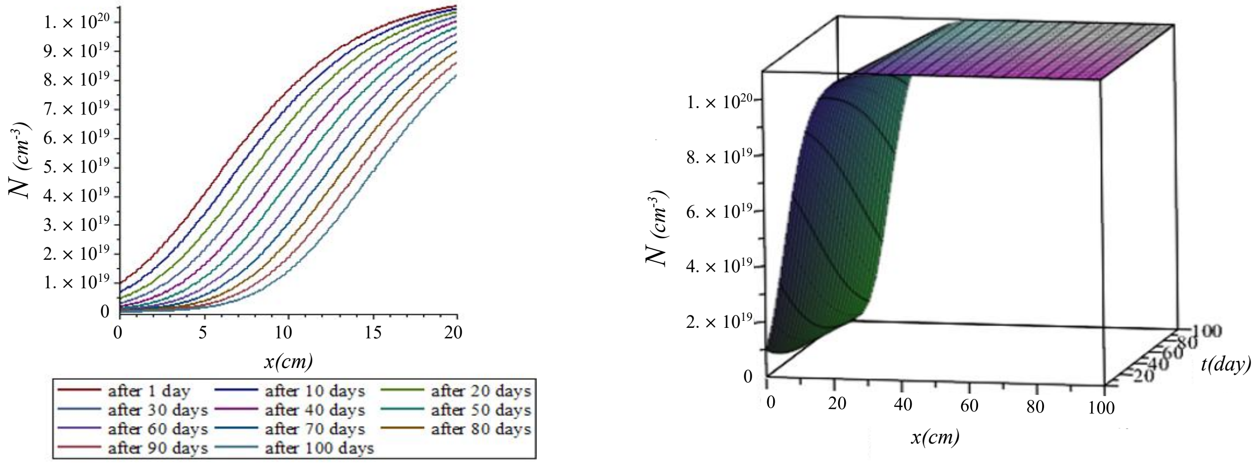
80
100
40

(d)



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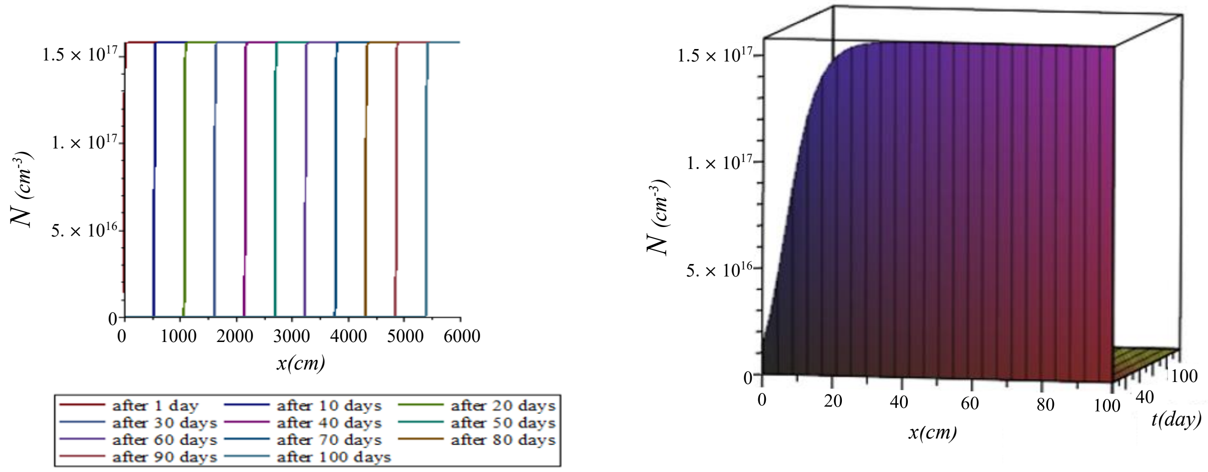
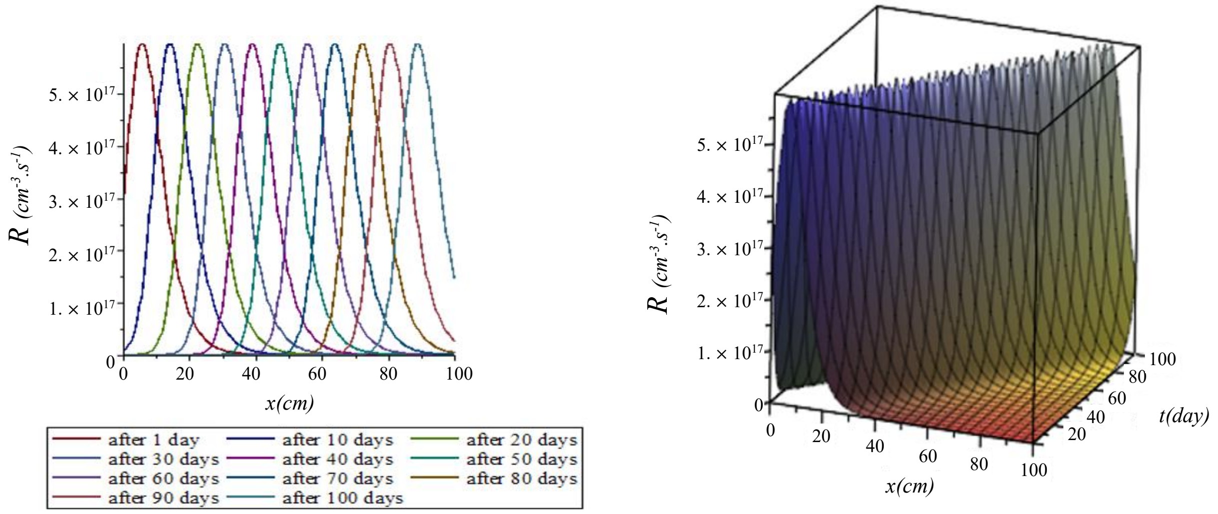
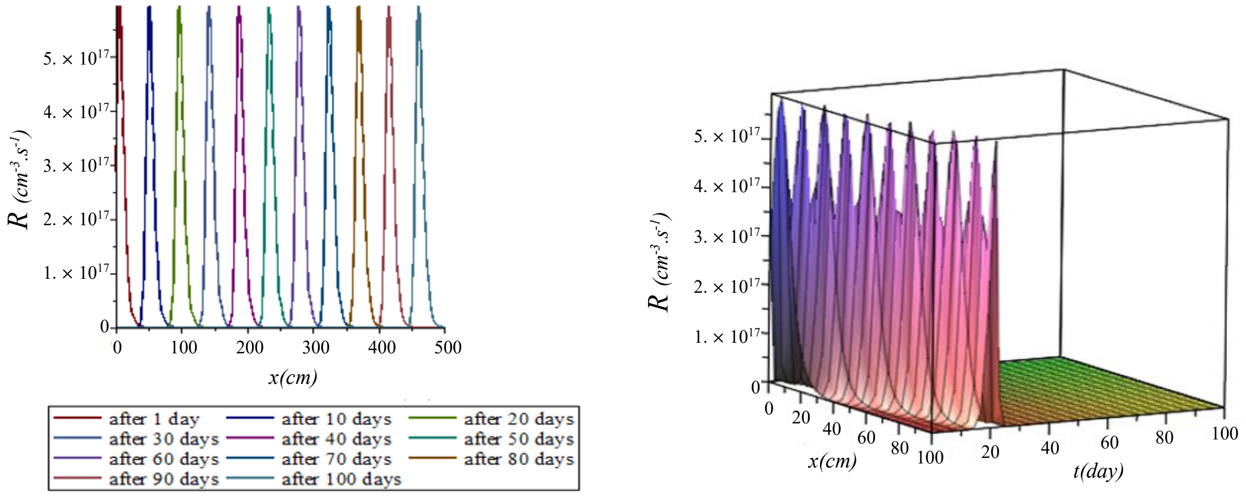


(f)

Fig. 3. Two and three-dimentional (2D and 3D) variations of the N for different neutron absorbing media: a) Boron (^{10}B), b) Cadmium (^{113}Cd), c) Samarium (^{149}Sm), d) Europium (^{151}Eu), e) Hafnium (^{177}Hf), and f) Gadolinium (^{157}Gd) as a function of position and time.



(a)


5. × 10^17
4. × 10^17
3. × 10^17
2. × 10^17
1. × 10^17
0
R (cm^-3.s^-1)
0
100
200
300
400
500
x(cm)
after 1 day
after 10 days
after 20 days
after 30 days
after 40 days
after 50 days
after 60 days
after 70 days
after 80 days
after 90 days
after 100 days
5. × 10^17
4. × 10^17
3. × 10^17
2. × 10^17
1. × 10^17
0
R (cm^-3.s^-1)
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x(cm)
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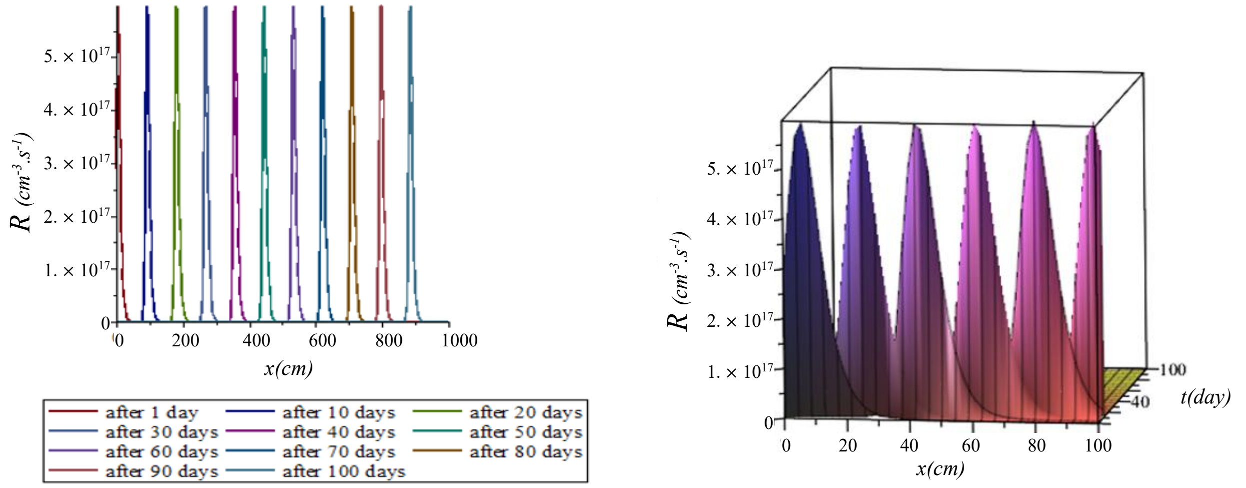
100
t(day)

(b)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4. × 10^17
3. × 10^17
2. × 10^17
1. × 10^17
0
R (cm^-3.s^-1)
0
200
400
600
800
1000
x(cm)
after 1 day
after 10 days
after 20 days
after 30 days
after 40 days
after 50 days
after 60 days
after 70 days
after 80 days
after 90 days
after 100 days
5. × 10^17
4. × 10^17
3. × 10^17
2. × 10^17
1. × 10^17
0
R (cm^-3.s^-1)
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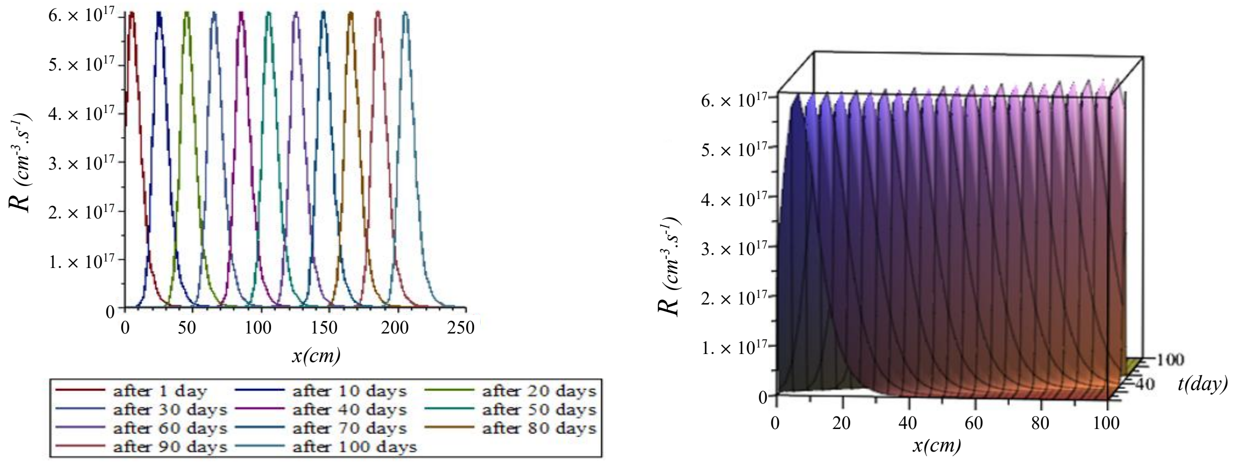
20
40
60
80
100
x(cm)
100
40
t(day)

(c)


6. × 10^17
5. × 10^17
4. × 10^17
3. × 10^17
2. × 10^17
1. × 10^17
0
R (cm^-3.s^-1)
0
50
100
150
200
250
x(cm)
after 1 day
after 10 days
after 20 days
after 30 days
after 40 days
after 50 days
after 60 days
after 70 days
after 80 days
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after 100 days
6. × 10^17
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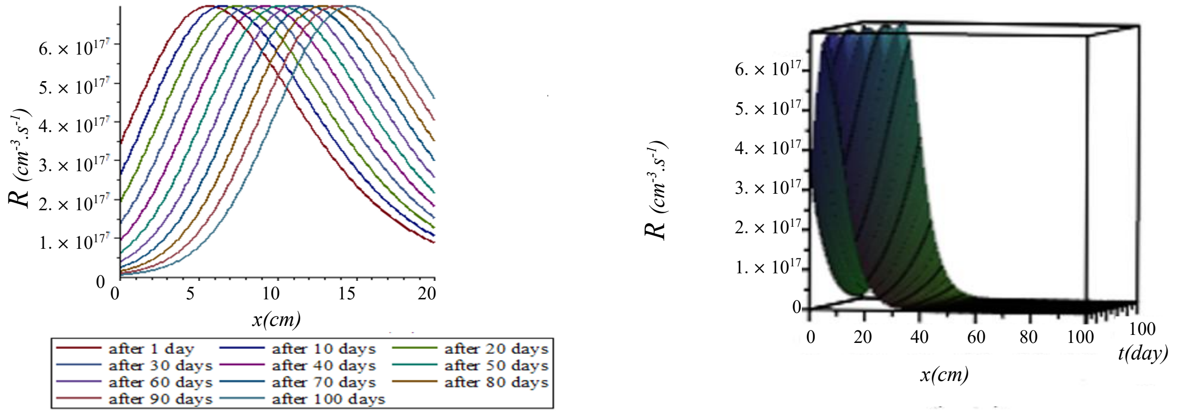
t(day)

(d)



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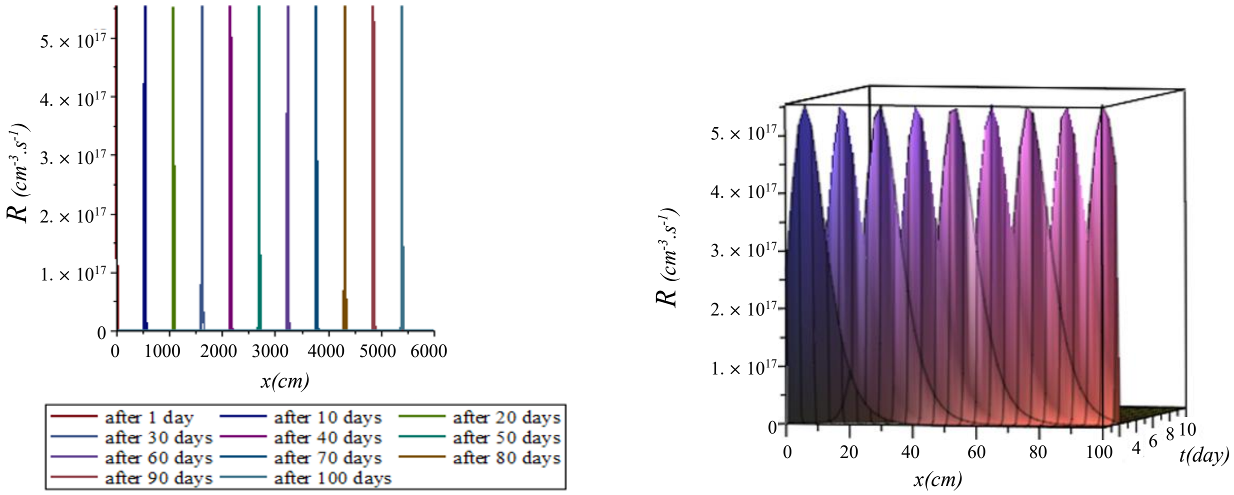

(f)

Fig 4. Two and three-dimentional (2D and 3D) variations of the R for different neutron absorbing media: a) Boron (^{10}B), b) Cadmium (^{113}Cd), c) Samarium (^{149}Sm), d) Europium (^{151}Eu), e) Hafnium (^{177}Hf), and f) Gadolinium (^{157}Gd)) as a function of position and time.

These figures can be described based on three different parameters: a) the Time of Transient (TOT) (the time required for the burn wave to reach the asymptotic form), b) the Length of Transient (LOT) (the length the wave travels to reach the stationary state), and c) the Steady State (SS) phase parameters (consisting of two basic parameters: 1) the wave velocity which is a value estimated by the net input neutron flux for each initial nuclide density. 2) the width of the reaction velocity region, which is described by FWHM). Based on these figures, these parameters have been determined for these selected NAs and their numerical values have been compared with each other in Table 3. Notice that, for each diffusive medium, the characteristics of time and diffusion length will be different due to their abundance and σ_a (absorption cross-section). Therefore, the numerical values of TOTs and LOTs can be described in terms of characteristics time and diffusion length.

Table 3. Characteristics of our obtained results for neutron burnup SW in different selected NAs in D_2O diffusive medium.

Element	^{10}B	^{113}Cd	^{149}Sm	^{151}Eu	^{177}Hf	^{157}Gd
Initial absorber density (atoms/cm^3)	1.04×10^{19}	19×10^{17}	9.8×10^{17}	44×10^{17}	11×10^{19}	1.58×10^{17}
Velocity analytical (cm/day)	0.8	4.5	8.8	2	0.1	54.7
LOT (cm)	13.8	13.9	13.8	13.8	13.6	13.7
Diffusion length (cm)	4.6	4.6	4.6	4.6	4.6	4.6
LOT per Diffusion length	3	3	3	3	3	3
TOT per Characteristics time	3.1	3.1	3.1	3.1	3.1	3.1
FWHM (cm)	13.0	13.0	12.9	12.9	12.7	13.8
FWHM per Diffusion length	2.81	2.81	2.81	2.81	2.81	2.81
Velocity simulation (cm/day)	0.8	4.6	8.8	2	0.08	55

Following results are obtained from Table 3: The calculated speed of the burnup wave is independent of the spreading coefficient according to the prediction; When the diffusion coefficient is enhanced, the LOT also grows. It was found that LOT per diffusion length does not change; The TOT increases by increasing the diffusion coefficient; The FWHM of the burnup SW enhances with the diffusion coefficient.

CONCLUSION

In this mathematical modeling research, we investigated the behavior of solitary waves in a uniform medium of different neutron absorbing materials and a heavy water diffusive medium. Our investigation may lead to the development of burnup SW in a diffusive medium by different NAs and the description of the properties of solitons. To achieve this goal, we considered the selected geometry as a finite slab. Due to its 1D geometry, the slab medium is very useful for explaining the concepts of analyzing the behavior of SWs. In a pure absorbing sample, neutrons scattering by nuclides is negligible. However, if we need to strengthen the speed of the burnup SW, the density of the absorbing material should be diluted by adding materials that do not absorb neutrons. Heavy water is one of the diffusive materials. Our studies were carried out on six NA media such as boron carbide, cadmium, samarium, europium, hafnium and gadolinium. It is important to mention that in all these media, we have considered the initial neutron input flux to be the same and equal to 10^{14} $cm^{-2}.\ s^{-1}$ in all cases. Our numerical calculations indicate that the speed of the burnup wave remains unchanged as the diffusion coefficient increases. However, the diffusion length and subsequent LOT increase. It is observed that the ratio of LOT and diffusion length remains constant. Additionally, increasing the diffusion coefficient leads to an increase in TOT, but the ratio between TOT and the characteristic time remains unchanged. This research provides new parameters and methods to describe the SW propagation and its relationship with the inherent parameters of the reactor. It expresses the relationship between LOT and TOT, and wave speed with absorption cross-section, mean free path, and characteristics time. The following suggestions are made for further study: (1) This study should be expanded and generalized for different elements. (2) The elements under study should be evaluated using the aforementioned parameters along with other factors such as economic factors and their chemical and physical structure for using in research nuclear reactors. (3) Nuclear fuel system designers can make decisions based on the values of the mentioned parameters, and the practical application of the burnup SW.

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AUTHOR CONTRIBUTION

S. N. Hosseinimotlagh and A. Shakeri equally contributed as the main contributors of this paper. All authors read and approved the final version of the paper.

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