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Pseudospin Symmetry of the Position-Dependent Mass Dirac Equation for the Hulthén Potential and Yukawa Tensor Interaction

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ABSTRACT

This paper presents approximate analytical solutions of the Dirac equation for the Hulthén potential with position-dependent mass within the framework of pseudospin symmetry limit using the Nikiforov-Uvarov method. The results showed the relativistic energy spectrum and the corresponding un-normalized wave function expressed in terms of the Jacobi polynomials.

INTRODUCTION

The Dirac equation with position-dependent mass (PDM) formulation has attracted a lot of interest in recent years because of its application in particle, nuclear, semiconductor and condensed matter physics [1]. In more precise words, PDM quantum systems have been found to be very useful in the analysis of microstructures such as quantum liquids [2,3], quantum dots [4], quantum wells [5], and semiconductor heterostructures [6]. Different authors have investigated the PDM for nonrelativistic quantum mechanics using various techniques [7,8]. The relativistic Dirac equation with PDM has been used to analyze heavy atoms and heavy ion doping [9]. In the Dirac equation, the pseudospin symmetry occurs when the magnitude of the attractive Lorentz scalar potential *S(r)* and repulsive vector potential *V(r)* are nearly equal but opposite in sign [10-12]. The tensor interaction was introduced into the Dirac equation by the transformation $\vec{p} \to \vec{p} - im\omega\beta.\hat{r}U(r)$ with spin-orbit coupling term being added to the Dirac Hamiltonian [13]. In most of studies, due to the mathematical structure of the problem, the tensor interaction is considered as Coulomb-like[14-15] or Cornell interactions. Hassanabadi et al. were the first who introduced the Yukawa tensor interaction in the Dirac theory and reported the corresponding approximate analytical solution [16].

From the mathematical point of view, the Dirac equation possesses an exact analytical solution only for a few well-known potentials. In the three spatial dimensions, this is due to the inverse square term appearing in the Hamiltonian. In the past years, a variety of polynomial and exponential-type potentials have been studied within the framework of the equation. In some of them, the tensor interaction is present and its effect on the degeneracy behavior of the system is investigated [17-21]. The main purpose of this article is to investigate the Dirac equation with a vector and scalar Hulthén potential for spin-1/2 particles and to obtain the approximate analytical solutions for an arbitrary spin-orbit coupling quantum number κ.

The Hulthén interaction is one of the successful short-range potentials [22] which behaves like a Coulomb potential when $r \to 0$. The potential has successfully accounted for some of the existing data in nuclear, particle, atomic, condensed matter, and chemical physics and has therefore been the subject of some related works in both nonrelativistic and relativistic regimes [23-26]. The Hulthén potential is a special case of the Manning–Rosen potential. In addition, we can compare the potential with the general Möbius-square form of the Eckart potential by appropriately choosing the parameters. The potential has the form [22]

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$$V(r) = -V_0 \frac{e^{-2\alpha r}}{\left(1-e^{-2\alpha r}\right)}, \quad V_0 = 2\alpha z \qquad (1)$$

where V_0, α and z are three real parameters and represent the strength, the screening range of the potential and the atomic number, respectively. In this paper, we consider the Hulthén potential with PDM besides a Yukawa tensor potential.

EXPERIMENTAL METHODS

The parametric nikiforov-uvarov method

Within this section, we will introduce the simple but powerful Nikiforov-Uvarov (NU) technique which has solved many important problems in quantum mechanics [27,28]. According to the NU method, a second-order differential equation of the form

$$\left\{\frac{d^2}{dr^2} + \frac{c_1 - c_2 s}{s(1-c_3 s)}\frac{d}{ds} + \frac{1}{s^2(1-c_3 s)^2}\left(-\xi_1 s^2 + \xi_2 s - \xi_3\right)\right\}\psi(s) = 0 \qquad (2)$$

has the solutions [29,30]

$$c_3 n - 2nc_5 + (2n+1)\left[\sqrt{c_9} + c_3\sqrt{c_8}\right] + n(n-1)c_3 + c_7 + 2c_3 c_8 + 2\sqrt{c_8 c_9} = 0, \qquad (3)$$

$$\psi(s) = s^{c_{12}}\left(1-c_3 s\right)^{c_{13}} P_n^{(c_{10},c_{11})}(1-2c_3 s), \qquad (4)$$

where $P_n^{(\alpha,\beta)}(s)$ are the Jacobi polynomials and

$$c_4 = \frac{1}{2}(1-c_1), c_5 = \frac{1}{2}(c_2 - 2c_3), c_6 = c_5^2 + \xi_1,$$
$$c_7 = c_4 c_5 - \xi_2, c_8 = c_4^2 + \xi_3, c_9 = c_3 c_7 + c_3^2 c_8 + c_6,$$
$$c_{10} = c_1 + 2c_4 + 2\sqrt{c_8}, c_{11} = c_2 - 2c_5 + 2\left(\sqrt{c_9} + c_3\sqrt{c_8}\right),$$
$$c_{12} = c_4 + \sqrt{c_8}, c_{13} = c_5 - \left(\sqrt{c_9} + c_3\sqrt{c_8}\right). \qquad (5)$$

Dirac equation with a tensor coupling

Dirac equation with a tensor potential $U(r)$ is written as $(\hbar = c = 1)$ [31]

$$\left[\vec{\alpha}.\vec{p} + \beta(M + S(r) - i\beta\vec{\alpha}.\hat{r}\,U(r)\right]\psi(r) = \left[E - V(r)\right]\psi(r) \qquad (6)$$

where E is the relativistic energy of the system, $\vec{p} = -i\vec{\nabla}$ is the three-dimensional momentum operator and M is the mass of the fermionic particle. $\vec{\alpha}, \beta$ are the 4×4 Dirac matrices given as

$$\vec{\alpha} = \begin{pmatrix} 0 & \vec{\sigma}_i \\ \vec{\sigma}_i & 0 \end{pmatrix}, \beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \qquad (7)$$

where I is a 2×2 unit matrix and $\vec{\sigma}_i$ are the Pauli three-vector matrices defined as

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \qquad (8)$$

For a spherical nuclei, the total angular momentum $\vec{J}$ and spin-orbit operator $K = -\beta(\sigma.\vec{L} + 1)$, where σ denotes the Pauli matrices and $\vec{L}$ is the orbital angular momentum operator, commute with Dirac Hamiltonian. The eigenvalues of the spin-orbit coupling operator are $\kappa = (j + \frac{1}{2}) > 0, \kappa = -(j + \frac{1}{2}) < 0$ for unaligned $j = l - \frac{1}{2}$ and aligned spin $j = l + \frac{1}{2}$ cases, respectively. The set $\left(H, K, J^2, J_z\right)$ forms a complete set of conserved quantities. Thus, we can write the spinors as [32]

$$\psi_{n\kappa}(r) = \frac{1}{r}\begin{pmatrix} F_{n\kappa}(r)Y_{jm}^{l}(\theta,\varphi) \\ iG_{n\kappa}(r)Y_{jm}^{\tilde{l}}(\theta,\varphi) \end{pmatrix} \qquad (9)$$

where $F_{n\kappa}(r)\,and\,G_{n\kappa}(r)$ represent the upper and lower components of the Dirac spinors, respectively. $Y_{jm}^{l}(\theta,\varphi), Y_{jm}^{\tilde{l}}(\theta,\varphi)$ are the spin and pseudospin spherical harmonics and m is the projection on the z-axis. With other known identities [33]

$$\left(\vec{\sigma}.\vec{A}\right)\left(\vec{\sigma}.\vec{B}\right) = \vec{A}.\vec{B} + i\vec{\sigma}.\left(\vec{A}\times\vec{B}\right),$$
$$\vec{\sigma}.\vec{p} = \vec{\sigma}.\hat{r}\left(\hat{r}.\vec{p} + i\frac{\vec{\sigma}.\vec{L}}{r}\right) \qquad (10)$$

as well as

$$\left(\vec{\sigma}.\vec{L}\right)Y_{jm}^{\tilde{l}}(\theta,\varphi) = (\kappa-1)Y_{jm}^{\tilde{l}}(\theta,\varphi)$$
$$\left(\vec{\sigma}.\vec{L}\right)Y_{jm}^{l}(\theta,\varphi) = -(\kappa-1)Y_{jm}^{l}(\theta,\varphi)$$
$$\left(\vec{\sigma}.\hat{r}\right)Y_{jm}^{l}(\theta,\varphi) = -Y_{jm}^{\tilde{l}}(\theta,\varphi)$$
$$\left(\vec{\sigma}.\hat{r}\right)Y_{jm}^{\tilde{l}}(\theta,\varphi) = -Y_{jm}^{l}(\theta,\varphi) \qquad (11)$$

we obtain the coupled equations [33],

$$\left(\frac{d}{dr}+\frac{\kappa}{r}-U(r)\right)F_{n\kappa}(r)=\left(M(r)+E_{n\kappa}-\Delta(r)\right)G_{n\kappa}(r), \quad (12)$$

$$\left(\frac{d}{dr}-\frac{\kappa}{r}+U(r)\right)G_{n\kappa}(r)=\left(M(r)-E_{n\kappa}+\Sigma(r)\right)F_{n\kappa}(r), \quad (13)$$

where,

$$\Delta(r)=V(r)-S(r), \quad (14)$$

$$\Sigma(r)=V(r)+S(r). \quad (15)$$

Eliminating one component in favor of the other, we obtain the second-order Schrödinger-like equations

$$\left\{\begin{array}{l}\dfrac{d^2}{dr^2}-\dfrac{k(k+1)}{r^2}+\dfrac{2kU(r)}{r}-\dfrac{dU(r)}{dr}-U^2(r)\\ -\left(\mu(r)+E_{nk}-\Delta(r)\right)\left(\mu(r)-E_{nk}+\Sigma(r)\right)\\ +\dfrac{\left(\dfrac{d\mu(r)}{dr}-\dfrac{d\Delta(r)}{dr}\right)\left(\dfrac{d}{dr}+\dfrac{k}{r}-U(r)\right)}{\left(\mu(r)+E_{nk}-\Delta(r)\right)}\end{array}\right\}F_{nk}(r)=0 \quad (16)$$

$$\left\{\begin{array}{l}\dfrac{d^2}{dr^2}-\dfrac{k(k-1)}{r^2}+\dfrac{2kU(r)}{r}+\dfrac{dU(r)}{dr}-U^2(r)\\ -\left(\mu(r)+E_{nk}-\Delta(r)\right)\left(\mu(r)-E_{nk}+\Sigma(r)\right)\\ +\dfrac{\left(\dfrac{d\mu(r)}{dr}+\dfrac{d\Sigma(r)}{dr}\right)\left(\dfrac{d}{dr}-\dfrac{k}{r}+U(r)\right)}{\left(\mu(r)+E_{nk}-\Sigma(r)\right)}\end{array}\right\}G_{nk}(r)=0 \quad (17)$$

with $\kappa(\kappa-1)=\tilde{l}(\tilde{l}+1), \kappa(\kappa+1)=l(l+1)$. The mathematical relation $\frac{d\mu(r)}{dr}=-\frac{d\Sigma(r)}{dr}=\frac{d\Delta(r)}{dr}$ is the necessary relation to obtain exact or approximate solutions for PDM problems [34].

The pseudospin symmetry limit

In the pseudospin symmetry limit, $\frac{d\Sigma(r)}{dr}=0$ or $\Sigma(r)=C_{ps}=const.$ [10-12]. Here, we consider [7,22]

$$\Delta(r)=-\frac{2z\alpha e^{-2\alpha r}}{\left(1-e^{-2\alpha r}\right)}, \qquad \mu(r)=\mu_0-\frac{2z\alpha e^{-2\alpha r}}{\left(1-e^{-2\alpha r}\right)} \quad (18)$$

besides the Yukawa tensor interaction [35]

$$U(r)=-V_1\left(\frac{e^{-\alpha r}}{r}\right), \quad (19)$$

where Z, μ_0,V_1 and $\alpha\alpha$ are Coulomb charge, the rest reduced mass, potential depth and range of the nucleon force, respectively [26]. Obviously, the corresponding equation is not exactly solvable. Consequently, to provide an analytical solution, we have to proceed on an approximate basis. Therefore, we introduce the approximations [36]

$$\frac{1}{r^2}\approx\frac{4\alpha^2e^{-2\alpha r}}{(1-e^{-2\alpha r})^2}, \quad (20)$$

$$\frac{1}{r^2}\approx\frac{4\alpha^2e^{-\alpha r}}{(1-e^{-2\alpha r})^2} \quad (21)$$

which are plotted in Fig. 1. The combination of recent equations as well as a change of variable of the form $s=e^{-2\alpha r}$, yields

$$\frac{d^2G_{nk}^{ps}}{dr^2}+\frac{(1-s)}{s(1-s)}\frac{dG_{nk}^{ps}}{ds}+\frac{1}{s^2(1-s)^2}\left[-Q_1^{ps}s^2+Q_2^{ps}s-Q_3^{ps}\right]G_{nk}^{ps}(s)=0 \quad (22)$$

with

$$Q_1^{ps}=\left(\frac{\varepsilon^2}{4\alpha^2}+V_1\left(V_1+\frac{1}{2}\right)+\frac{\beta}{4\alpha^2}\right),$$

$$Q_2^{ps}=\left(\frac{2\varepsilon^2}{4\alpha^2}-\kappa(\kappa-1)-\left(2\kappa-\frac{3}{2}\right)V_1+\frac{\beta}{4\alpha^2}\right), \quad (23)$$

$$Q_3^{ps}=\frac{\varepsilon^2}{4\alpha^2},$$

$$\varepsilon^2=\left(\mu_0-E_{n\kappa}^{ps}+C_{ps}\right)\left(\mu_0+E_{n\kappa}^{ps}\right), \beta=2z\alpha\left(\mu_0+E_{n\kappa}^{ps}\right).$$

Comparing Eqs. (22) and (2), we obtain the required parameters as

$$c_1=1,\xi_1=Q_1^{Ps}, \quad c_2=1,\xi_2=Q_2^{Ps}, \quad c_3=1,\xi_3=Q_3^{Ps}. \quad (24)$$

Eq. (5) determines the rest of the coefficients as

$$c_4=0, c_5=-\frac{1}{2}, c_6=\frac{1}{4}+Q_1^{ps},$$

$$c_7=Q_2^{ps}, c_8=Q_3^{ps}, c_9=Q_1^{ps}+Q_3^{ps}-Q_2^{ps}+\frac{1}{4}, c_{10}=1+2\sqrt{Q_3^{ps}},$$

$$c_{11}=2+2\left(\sqrt{\frac{1}{4}+Q_1^{ps}+Q_3^{ps}-Q_2^{ps}+\sqrt{Q_3^{ps}}}\right),$$

$$c_{12}=\sqrt{Q_3^{ps}}, c_{13}=-\frac{1}{2}-\left(\sqrt{\frac{1}{4}+Q_1^{ps}+Q_3^{ps}-Q_2^{ps}+\sqrt{Q_3^{ps}}}\right). \quad (25)$$

Using Eqs. (24) and (25), we can easily obtain the energy relation

$$n+\frac{(2n+1)}{2}+(2n+1)\left(\sqrt{Q_1^{ps}+Q_3^{ps}-Q_2^{ps}+\frac{1}{4}}+\sqrt{Q_3^{ps}}\right)+n(n-1)-Q_2^{ps}+2Q_3^{ps}$$
$$+2\sqrt{Q_3^{ps}\left(Q_1^{ps}+Q_3^{ps}-Q_2^{ps}+\frac{1}{4}\right)}=0, \quad (26)$$

and lower component of the wave function is

$$G_{nk}^{ps}(r)=N_{nk}\left(e^{-2\alpha r}\right)^{\sqrt{Q_3^{ps}}}\left(1-e^{-2\alpha r}\right)^{\frac{1}{2}+\sqrt{\frac{1}{4}+Q_1^{ps}+Q_3^{ps}-Q_2^{ps}}} \quad (27)$$
$$\times_2 F_1\left(-n,n+2\sqrt{Q_3^{ps}}+2\sqrt{\frac{1}{4}+Q_1^{ps}+Q_3^{ps}-Q_2^{ps}}+1;2\sqrt{Q_3^{ps}}+1;e^{-2\alpha r}\right).$$

The corresponding upper component can be simply obtained from Eq.(12), i.e.

$$F_{n\kappa}^{ps}(r)=\frac{1}{\left(M-E_{n\kappa}^{ps}+C_{ps}\right)}\left(\frac{d}{dr}-\frac{\kappa}{r}-V_1\left(\frac{e^{-\alpha r}}{r}\right)\right)G_{n\kappa}^{ps}(r). \quad (28)$$

RESULTS AND DISCUSSION

Approximate solutions of the Dirac equation for the Hulthén potential were obtained in the absence and the presence of the Yukawa tensor potential for various values of the quantum numbers n and κ. The bound-states are reported in table 1 under the condition of the pseudospin symmetry and we can clearly see the degenerate behavior between the bound-states which ischanged when the Yukawa tensor interaction is present. In Fig. 2, it is shown the effects of the α-parameter on the bound-states in the presence of the tensor potential (V_1 = *0.5)* where it is seen that for increasing α, the bound-states become more bounded. In Fig. 3, the behavior of energy vs. C_{ps} for pseudospin symmetry limit was obtained it can be seeing that the pseudospin bound-states become less bounded with increasing C_{ps}. Figure 4 represents the dependence of the bound-state energy levels on the potential parameter *z*. It is seen that the pseudospin bound-states become more bounded with increasing *z*. In Figs. 5 and 6, the effects of the tensor interaction parameters V_1 and μ_0 on the bound-states in view of the pseudospin symmetry limit were determined respectively. Figure 5 shows that the magnitude of the energy difference between the degenerate states increases as *H* increases. In Fig. 6, it can be seen that the bound-states become less bounded with increasing μ_0. In Fig. 7, the wavefunctions are plotted for vanishing and existing tensor where we observe that the tensor interaction only affects the shape of the wave functions and does not change the node structure of the radial upper and lower components of the Dirac spinors.

Table 1. Energies in the Pseudospin Symmetry Limit for α = *0.01,* μ_0= *1fm*$^{-1}$, *z=-1,* C_{ps} = *-5*

$\tilde{\ell}$	$n,\kappa<0$	(ℓ,j)	$E_{n\kappa}^{ps}(fm^{-1})$ $(V_1=0)$	$E_{n\kappa}^{ps}(fm^{-1})$ $(V_1=0.5)$	$n-1,$ $\kappa>0$	$(\ell+2,$ $j+1)$	$E_{n\kappa}^{ps}(fm^{-1})$ $(V_1=0)$	$E_{n\kappa}^{ps}(fm^{-1})$ $(V_1=0.5)$
1	**1,-1**	$1S_{\frac{1}{2}}$	**-3.928349615**	**-3.893285734**	**0,2**	$0d_{\frac{3}{2}}$	**-3.928349615**	**-3.949025469**
2	**1,-2**	$1P_{\frac{3}{2}}$	**-3.963160651**	**-3.949025469**	**0,3**	$0f_{\frac{5}{2}}$	**-3.963160651**	**-3.972408645**
3	**1,-3**	$1d_{\frac{5}{2}}$	**-3.979367223**	**-3.972408645**	**0,4**	$0g_{\frac{7}{2}}$	**-3.979367223**	**-3.984191529**
4	**1,-4**	$1f_{\frac{7}{2}}$	**-3.988044887**	**-3.984191529**	**0,5**	$0h_{\frac{9}{2}}$	**-3.988044887**	**-3.990805160**
1	**2,-1**	$2S_{\frac{1}{2}}$	**-3.963160651**	**-3.949025469**	**1,2**	$1d_{\frac{3}{2}}$	**-3.963160651**	**-3.972408645**
2	**2,-2**	$2P_{\frac{3}{2}}$	**-3.979367223**	**-3.972408645**	**1,3**	$1f_{\frac{5}{2}}$	**-3.979367223**	**-3.984191529**
3	**2,-3**	$2d_{\frac{5}{2}}$	**-3.988044887**	**-3.984191529**	**1,4**	$1g_{\frac{7}{2}}$	**-3.988044887**	**-3.990805160**
4	**2,-4**	$2f_{\frac{7}{2}}$	**-3.993092019**	**-3.990805160**	**1,5**	$1h_{\frac{9}{2}}$	**-3.993092019**	**-3.994756044**
1	**3,-1**	$3S_{\frac{1}{2}}$	**-3.979367223**	**-3.972408645**	**2,2**	$2d_{\frac{3}{2}}$	**-3.979367223**	**-3.984191529**
2	**3,-2**	$3P_{\frac{3}{2}}$	**-3.988044887**	**-3.984191529**	**2,3**	$2f_{\frac{5}{2}}$	**-3.988044887**	**-3.99080516**
3	**3,-3**	$3d_{\frac{5}{2}}$	**-3.993092019**	**-3.990805160**	**2,4**	$2g_{\frac{7}{2}}$	**-3.993092019**	**-3.994756044**
4	**3,-4**	$3f_{\frac{7}{2}}$	**-3.996160182**	**-3.994756044**	**2,5**	$2h_{\frac{9}{2}}$	**-3.996160182**	**-3.997182574**

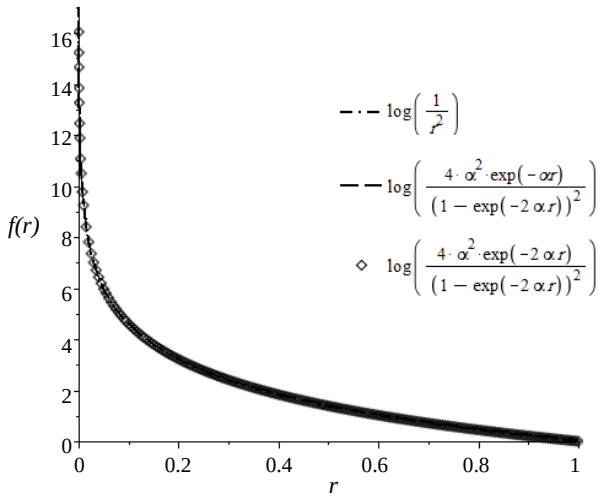

Fig. 1. $\log(\frac{1}{r^2})$)and its approximations for α = 0.01.

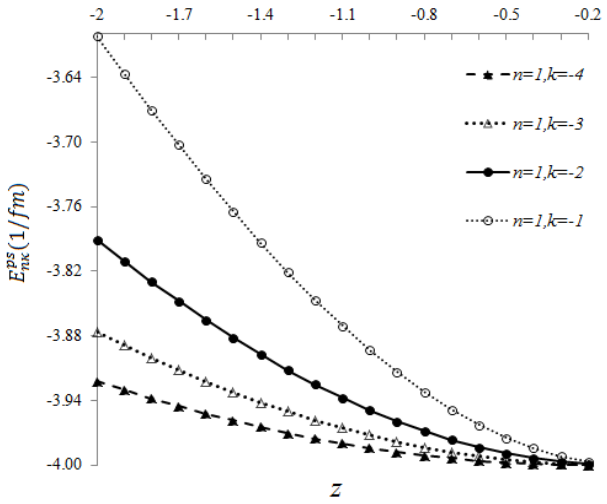

Fig. 4. Energy vs. z for pseudospin Symmetry limit for α = 0.01, μ_0= 1fm^{-1}, C_{ps} = -5, V_1 = 0.5.

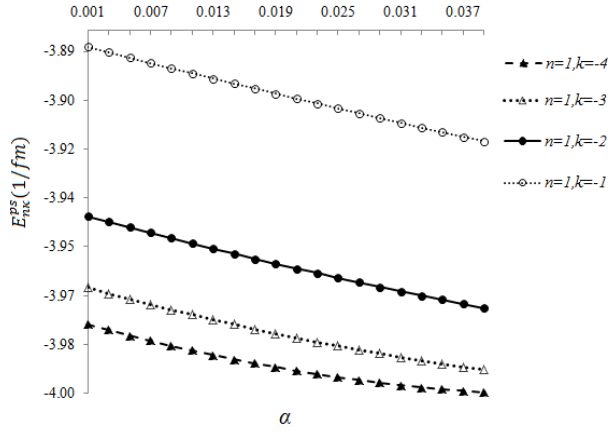

Fig. 2. Energy vs. α for pseudospin Symmetry limit for μ_0= 1fm^{-1}, z=-1, C_{ps} = -5, V_1 = 0.5.

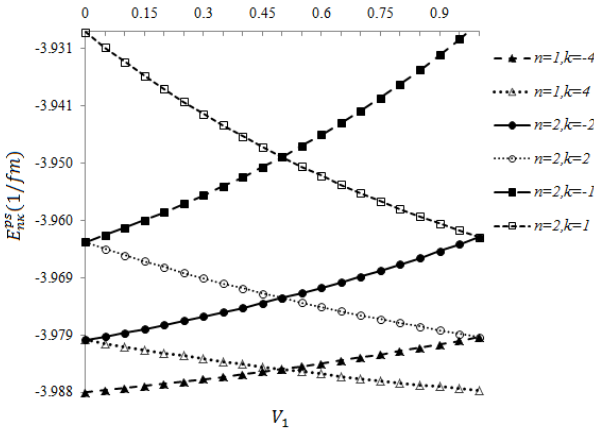

Fig. 5. Energy vs. V_1 for pseudospin Symmetry limit for α = 0.01, μ_0= 1fm^{-1}, z=-1, C_{ps} = -5.

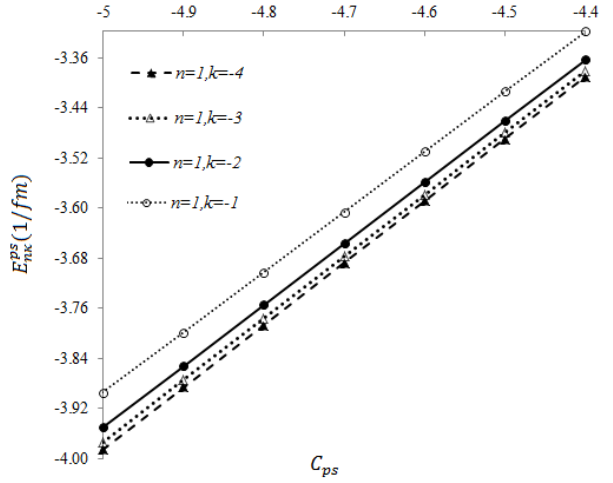

Fig. 3. Energy vs. C_{ps} for pseudospin Symmetry limit for α = 0.01, μ_0= 1fm^{-1}, z=-1, V_1 = 0.5.

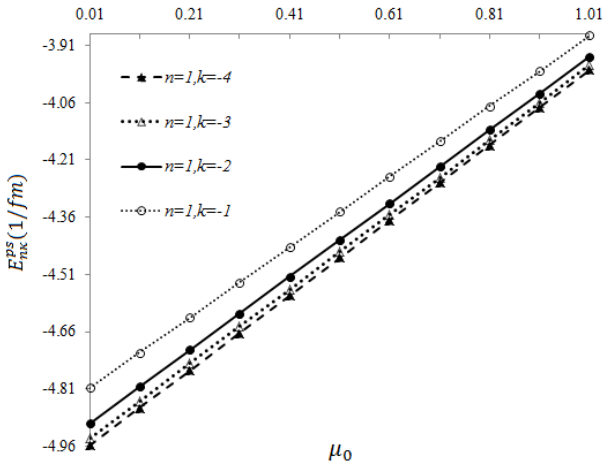

Fig. 6. Energy vs. μ_0 for pseudospin Symmetry limit for α = 0.01, z=-1, C_{ps} = -5, V_1 = 0.5.

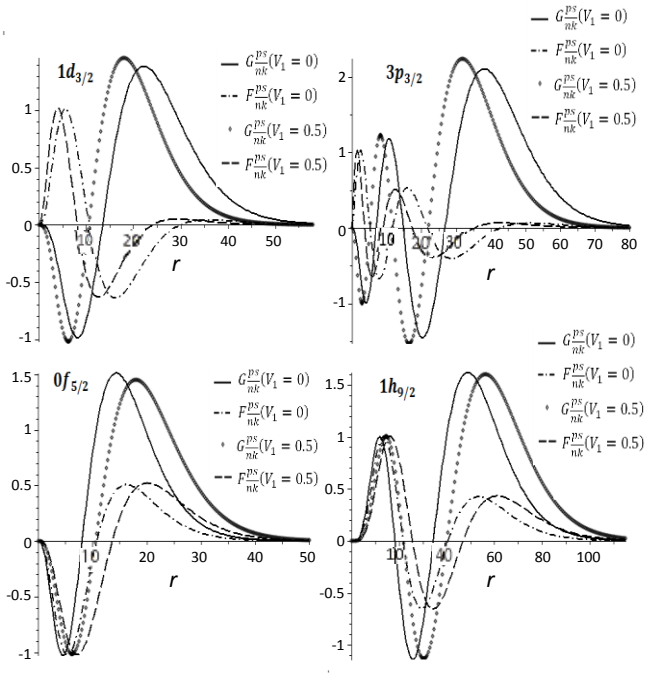

Fig. 7. Wavefunction for Pseudospin Symmetry Limit for $\alpha = 0.01$, $\mu_0 = 1fm^{-1}$, $z=-1$, $C_{ps} = -5$, $V_1 = 0.5$.

CONCLUSION

In this paper, was obtained the approximate analytical solutions of the Dirac equation for the Hulthén potential within the framework of PDM and in the presence of a Yukawa tensor interaction term within the framework of pseudospin symmetry limit using the NU method. The results show that the energy eigenvalues and corresponding lower and upper wave functions in terms of the Jacobi polynomials. Finally, this work can be extended to others potentials model [7,8,34] which has many applications in physics and related fields.

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